Modelling Sustainable Green Growth*

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Abstract

A global shift towards designing net-zero carbon policy for sustainable growth is emerging. Using the lens of an endogenous growth model, this paper explores three key aspects of such policy (i) growth with a renewable biased technology, (ii) public investment in renewable resources, and (iii) a carbon tax to promote the use of renewables. In our model, the final output is produced with two inputs, renewable and nonrenewable capital. Use of nonrenewable capital causes depreciation of renewables via emissions and externalities that pose a deadweight loss by depressing growth. The carbon tax encourages substitution of nonrenewable capital with renewable capital towards a net-zero carbon target. We show that (i) the carbon tax will need to be higher if the technology allows less substitution of renewable with nonrenewable capital, (ii) renewable economies enjoy higher growth and less need for carbon taxation, and (iii) greater efficiency in emissions abatement promotes long run growth by reducing the deadweight loss.

Key words: Renewable resources, sustainability, carbon tax, endogenous growth, resource substitution.
JEL Classifications E1, O3, O4, Q2

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1. Introduction

A global shift towards developing policies for a transition involving renewable resources and sustainable growth is taking shape. In the EU, this shift manifests itself in the adoption of the European Green Deal for transition towards net-zero economy by 2050. In the UK as well, the flagship policy of Industrial Strategy aims to boost growth through the promotion of cost effective low carbon technologies. While the net-zero strategy lays out the goal of a clean growth, it is less clear about the links between economic, environmental, and resource aspects and their trade-offs. The challenge emanates from a long standing theoretical and policy debate on whether growth is possible without exhausting natural resources. The proponents of strong sustainability (e.g, Daly, 1997; Ayres, 2007), hold the view that there is limited scope for such substitution. Sustainabilty is, however, not a binary concept. Solow (1974) and Nordhaus and Tobin (1972) take a weak sustainability view that suggests some degree of substitution is possible between these renewable and nonrenewable capital.[1] The crux of the debate boils down to how or to what extent renewable and nonrenewable resources are substitutable to achieve the net-zero carbon target of the UK government as well as EU. If so, what policy instruments could accomplish this task?

Achieving a target of net-zero carbon and sustaianable green growth faces two immediate challenges. First, the businesses mostly use carbon intensive technology and do not necessarily internalize the damage to the aggregate economy. Greenhouse

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[1] For a recent survey on the sustainability aspects of growth, see Cerkez (2018).

gas emissions cause global warming and climate change and can erode the renewable capital base of the economy. Second, there are limits to renewability imposed by the planet's resources because renewables need a massive amount of natural resources. In our model, there are two policy instruments to mitigate these two problems. First, we introduce a corrective Pigovian carbon tax which balances the social and private marginal cost of emission. Second, we add public investment to augment and enrich renewable. There are two types of such green public investment, namely (i) augmenting the stock of renewables (e.g. building more windpower and power plants), and (ii) various forms of pollution abatement investments. This public investment is financed by the carbon tax revenue.

We model sustainable growth using a prototype AK type endogenous growth model as in Rebelo (1991).[2] In our model, the broad based composite capital consists of renewable and nonrenewable capital. Since renewable capital draws on the planet's natural resources, we also label it *green capital.* The nonrenewable capital is fossil fuel intensive. The underlying production technology is kept broad to allow for different degrees of substitution between these two types of capital. The strong sustainability approach to growth arises as a special case in our setting when the production function has near zero substitutability between these two types of capital. Since our focus is on long run sustainable growth with a net-zero emission target, we abstract from short run dynamics and adjustment costs of changing the composition of capital.

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[2] Stokey (2005) also used this type of model to address the issue of pollution abatement. For an excellent survey of the empirics of endogenosus growth models, see Capoloupo (2009).

Several important insights become evident from our stylized model. First, the optimal carbon tax is progressively higher when substitution of renewable by nonrenewable capital is restricted because of difficulty of decarbonising a production process. Second, the need for a carbon tax is less in economies with a more renewable or green intensive technology. Higher ratio of renewable to nonrenewable capital boosts the marginal product of nonrenewable capital because of its relative shortage and through this channel boosts economic growth. Third, a pollution abatement technology can achieve a net-zero emission target. Such pollution abatement is one of the cornerstones of UK's green industrial strategy. Such an abatement technology can take various forms including carbon capture, usage and storage system (CCUS).[3] We introduce a pollution abatement technology in an extended model. In this scenario, a combination of carbon tax, public investment in abatement and green capital replenishment could restore the Pareto optimal proportion of nonrenewable to green capital. Greater efficiency in pollution abatement boosts the long run growth, and lowers the carbon tax. Our results are timely and topical when in the post-Covid era, most advanced industrial economies struggle to strike a balance between growth and environmental policy with a zero emissions target.

The remainder of the paper is organized as follows. Section 2 connects to the related literature. Section 3 sets up a social planning problem which characterizes the socially optimal sustainable growth with optimal public and private investments in green and nonrenewable capital. Section 4 develops a model of a decentralized

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[3] CCUS technology is being discussed in recent days. Norway is reconsidering such CCS technology after its initial attempt was abandoned due to excessive cost. Japan has added CCS in its 2050 net-zero climate plans (Financial Times, April 25, 2021).

economy with a benevolent government, which replicates the allocation of the social planning optimum. Section 5 reports illustrative simulation results of our baseline model. Section 6 extends the model to include public sector investment in pollution abatement. Section 7 concludes.

2. Connections to literature

Our paper relates to a strand of literature on the effect of environmental tax on economic growth. Forster (1973) analyzes optimal capital accumulation in the presence of pollution. His framework was subsequently extended by Gruver (1976), Luptacik and Schubert (1982), and Siebert (1987). Gradus and Smulders (1993) present a comprehensive analysis of the environmental policy in terms of pollution abatement.[4] Cohen et al. (2019) review the literature on the substitution of clean and polluting technologies including the evidence from energy intensive industries and agriculture and conclude that, in practice, this substitution has been limited. This implies that considerable technological progress is required to increase this substitutability in the future as part of the decarbonisation efforts. Using a learning by doing technology and pollution distaste in the utility function, Michel and Rotillon (1995) argue that capital should be mostly taxed. A feature of their model is that a social optimum that internalizes pollution distaste might lead to a zero long run growth unless there is strong consumption compensation for pollution distaste.

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[4] Using a two-good general equilibrium model, Hollady et al. (2018) examine the effect of environmental regulation on the emissions leakage in the presence trade frictions They analyze the effect of an emissions tax but abstract from capital accumulation, growth and production based externality from emission which is our primary focus in this paper.

Stokey (1998) uses a pollution technology embedded in a simple AK model of endogenous growth and analyze limits to growth. Her model makes important predictions about limits to growth and the efficacy of tax and voucher schemes as opposed to direct regulations using social planning model as a benchmark. Our model has a similar spirit but we explicitly model the evolution of renewable capital and show how a Pareto optimal sustainable growth can be implemented using carbon tax.

Gars and Olovsson (2019) document that countries using fossil fuel instead of biofuel embark on a higher growth path and develop an endogenous growth model that explains this. In many of these papers, a common theme is a trade-off between environmental protection and growth. Fankhauser and Tol (2005) analyze the growth effect of climate change using a one sector neoclassical growth model. While they analyze the effect of climate change on growth using a growth model with physical capital only, our model has renewable and non-renewable capital and we show explicitly how a climate shock impacts the erosion of renewable capital. Bretschger and Vinogradova (2019) develop a dynamic growth model to analyze the optimal policy response to climate shocks with a single physical capital. Their focus is on the effect of climate uncertainty and thus the model is stochastic while our focus is on sustainable growth with complementarity between renewable and nonrenewable capital.

Our model also connects to the Dynamic Integrated Model of Climate and the Economy (DICE) of Nordhaus (2018) that uses Cass-Koopmans growth models with forward looking agents to analyze the economic effects of climate change. In the DICE model, many aspects of the environment are mapped into temperature as a single state variable. Such a mapping is motivated by natural science modules where fossil fuel emissions lead to higher temperature due to greenhouse effect. In the spirit of the DICE model, we characterize the production of a single composite final good with labour, carbon intensive nonrenewable capital and natural capital. The natural capital depreciates due to greenhouse effects of carbon intensive nonrenewable capital.

The technology of final goods production in our model is similar to Gars and Olovsson (2019). We consider two types of capital, nonrenewable (fossil fuel intensive) and renewable capital (biofuel intensive) in our production function.[5] The novelty of our setting is that we model the effects of climate shock on the aggregate economy when the stock of renewable capital erodes due to carbon emissions from fossil fuel intensive capital.

Our market economy model replicates the efficient allocation using the tax-subsidy mechanism as an environmental policy instrument. Alternatively, one can introduce pollution permits as an environmental policy instrument where a fixed number of pollution permits are auctioned off by the government to pollutant firms. Invoking Coase theorem, one can hope to achieve efficient allocations. We do not take this avenue because of the limitations of this approach due to the free rider problem pointed out by Chari and Jones (2000).

Finally, our paper sits at the intersection of two related aspects of the climate

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[5] Our model is similar in spirit to a class of models with private and public capital as in Futagami et al. (1993),Glomm and Ravikumar (1997), Turnovsky (1997) amongst others. However, their focus is primarily on public capital while we focus on renewable capital and carbon tax to finance the public investment in renewable with a net zero target.

change literature both laid out in detail by Stern (2007). The first stream is on the mitigation aspect and the second is on adaptation. Here we focus on carbon tax as a mitigation instrument and device an optimal tax which balances growth with the environment. We also formulate a pollution abatement technology which can balance the trade-off between environmental policy and economic growth.

3. Sustainability of growth as a social planning problem

The economy produces the final output (Y_t) with broad based capital (K_t) and a unit raw labour with a linear technology as in Rebelo (1991):

$$Y_t = AK_t \tag{1}$$

where A is a constant total factor productivity (TFP) term. The aggregate capital is composed of nonrenewable capital (K_t^p) and renewable or green capital (K_t^g) based on the following constant elasticity of substitution (CES) aggregation:

$$K_t = \left[(1-\nu)K_t^{p^{\varphi}} + \nu K_t^{g^{\varphi}}\right]^{1/\varphi} \tag{2}$$

with $0 < \nu < 1$,and $\varphi = (\sigma - 1)/\sigma$ where σ is the elasticity of substitution. Note that since σ is positive by construction $-\infty < \varphi < 1$. [6]

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[6] Our production function is similar in spirit to Gars and Olovsson (2019). In their model, the production of final goods requires the use of biofuel and fossil fuel which are produced with capital stocks different varieties. In our setting, we abstract from varieties and focus on a production function involving reproducible (biofuel intensive) capital and non-reproducible (fossil intensive) capital.

The nonrenewable capital evolves according to the linear depreciation rule:

$$K_{t+1}^p = (1 - \delta_p) K_t^p + I_t^p \tag{3}$$

where I_t^p is the level of private investment in nonrenewable capital and δ_p is its rate of depreciation.[7]

A benevolent social planner invests a share of final output, i_{yt}^g to replenish renewable capital by planting trees among other means.[8] The law of motion of renewable capital stock is given by:

$$K_{t+1}^g = (1 - \delta_{gt}) K_t^g + i_{yt}^g Y_t \tag{4}$$

Greater stock of non-renewable capital could cause erosion of renewable capital base of the economy (in the form of deforestation and climate change). Viewed from this perspective, we can call such an erosion *green erosion* of the economy. Keeping this in mind, we posit that this green erosion rate (δ_{gt}) is proportional to the ratio of non-renewable to renewable capital. In other words:

$$\delta_{gt} = \omega_t \frac{K_t^p}{K_t^g} \tag{5}$$

where ω_t is the single state variable which can be called a *climate shock* as the

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[7] Since the central interest of this paper is on sustainable growth, we ignore short run adjustment costs for changing these two types of capital. The balanced growth rate is unaltered by the presence of such adjustment cost.

[8] We represent the investment in man-made capital in level but green investment in rate. This distinction is crucial to justify a carbon tax rate in a decentralized economy.

temperature is in the DICE model of Nordhaus (2018). For our baseline model, we assume that ω_t is time invariant meaning $\omega_t = \overline{\omega}$ for all t and is exogenous as in Fankhauser and Tol (2005). A higher $\overline{\omega}$ in the present baseline model means a permanent global warming. Later on we partly endogenize ω_t via a pollution abatement technology.

Plugging (5) into (4), the law of motion of renewable capital reduces to:

$$K_{t+1}^{g} = \max(0, K_t^g - \overline{\omega} K_t^p + i_{yt}^g Y_t) \tag{6}$$

If $\overline{\omega}$ is too large, then the stock of renewable capital can approach zero. By setting the following upper bound on the net erosion of renewable capital, we rule out the possibility of negative renewable capital.

$$\overline{\omega} < \omega^* \tag{7}$$

where $\omega^*$is the minimum of the roots of $K_t^g - \overline{\omega} K_t^p + i_{yt}^g Y_t = 0$.[9]

The social planner determines a socially desirable sustainable growth that maximizes the welfare of a representative infinitely lived agent. Noting that C_t is the consumption of the agent at date t and β is a constant discount factor, formally the

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[9] One can rewrite this equation as

$$\overline{\omega} = (K_t^g / K_t^p) + A i_y^g \left[1 - \nu + \nu (K_t^g / K_t^p)^{\varphi}\right]^{1/\varphi}$$

where K_t^g / K_t^p and i_y^g are the steady state values given by (10) and (16). There could be multiple roots. We pick the minimum among them.

optimization problem is written as:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{8}$$

s.t.

$$C_t + I_t^p \leq (1 - i_{yt}^g) Y_t; \;\; K_0^g, \; K_0^p = \text{given} \tag{9}$$

and (1), (2), (3), (6), (9) and also the inequality constraint $i_{yt}^g \leq 1$. We do not impose any non-negativity constraint on either i_{yt}^g and I_t^p because we allow for disinvestment in both types of capital.

The interior solution is guaranteed by the upper bound on $\overline{\omega}$ and absence of non-negativity constraints on investment.[10]| The planner first chooses the time paths of non-renewable and renewable capital to equate the marginal product of man-made (MPK^p) with the marginal product of renewable capital (MPK^g), net of depreciation rates of both types of capital. In other words, the following static efficiency condition must hold:

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p \tag{10}$$

where

$$MPK_t^p = A(1-\nu)\left[(1-\nu) + \nu\left(\frac{K_t^g}{K_t^p}\right)^{\varphi}\right]^{\frac{1-\varphi}{\varphi}} = \Theta\left(\frac{K_t^g}{K_t^p}\right) \text{ (say)} \tag{11}$$

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[10] Note also i_{yt}^g cannot be unity because it means negative consumption The zero consumption is ruled out by the usual Inada condition.

and

$$MPK_t^g = A\nu \left[\nu + (1-\nu) \left(\frac{K_t^g}{K_t^p} \right)^{-\varphi} \right]^{\frac{1-\varphi}{\varphi}} = \Psi \left(\frac{K_t^g}{K_t^p} \right) \text{ (say)} \tag{12}$$

The derivation of the static efficiency condition (10) is in the appendix. Since there are no non-negativity constraints on these two types of investment and no adjustment costs, there is no transitional dynamics in this setting. We have the following proposition.

Proposition 1. *Based on the static efficiency condition (10), a unique ratio of renewable to non-renewable capital,* $\frac{K_t^g}{K_t^p}$ *exists. Higher* $\overline{\omega}$ *necessitates a higher ratio of renewable to nonrenewable capital to restore efficiency.*

Proof. It follows from the fact that $\Theta(0) = A(1-\nu)^{1/\varphi}$, $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$ and $\Psi(0) = \infty$, $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$. Thus, there exists a unique crossing point in the positive quadrant between $\Theta\left(\frac{K_t^g}{K_t^p}\right)$ and $\Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p$ schedules. Figure 1 demonstrates the existence of a unique K_t^g/K_t^p. ■

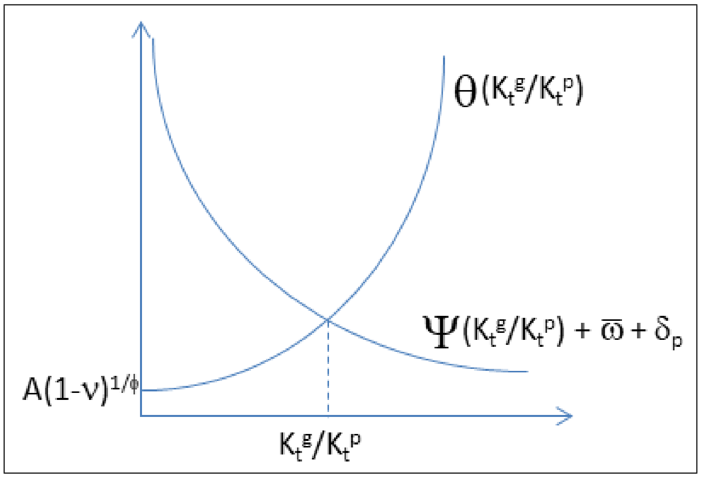

Figure 1: Existence of K_t^g/K_t^p

Using the implicit function theorem, and exploiting the fact that $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$ and $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$, it is straightforward to verify that

$$\frac{\partial(K_t^g/K_t^p)}{\partial\overline{\omega}} = \frac{1}{\left[\Theta'\left(\frac{K_t^g}{K_t^p}\right) - \Psi'\left(\frac{K_t^g}{K_t^p}\right)\right]} > 0 \tag{13}$$

The efficiency condition dictates that a permanent global warming (higher $\overline{\omega}$) requires more stringent quantity control of non-renewable capital by either divesting in non-renewable capital or investing in renewable capital. Either of these two actions or a combination of them boosts the ratio K_t^g/K_t^p. The social planner mandates a higher ratio of renewable to non-renewable capital when the environmental damage is higher. This can also be easily checked from Figure 1. Higher $\overline{\omega}$ makes the $\Psi(.) + \overline{\omega} + \delta_p$ shift out resulting in a higher equilibrium K_t^g/K_t^p.

The long run balanced growth rate (γ) must satisfy the following conditions:

$$1 + \gamma = \beta\left[1 + \Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{14}$$

$$1 + \gamma = \beta\left[1 + \Theta\left(\frac{K_t^g}{K_t^p}\right) - \delta_p - \bar{\omega}\right] \tag{15}$$

Since $\Psi'(.) < 0$, the implication is that a higher $\overline{\omega}$ unambiguously lowers the balanced growth rate via a rise in K_t^g/K_t^p. Therefore, growth is highest with zero erosion.

In the next step, the planner sets the optimal investment rate in green capital using (6) to ensure that the efficient balanced growth rate is achieved. Based on (6) such investment rate is given by:

$$i_y^g = \frac{\gamma + \overline{\omega}(K_t^g/K_t^p)^{-1}}{A\left[(1-\nu)(K_t^g/K_t^p)^{-\varphi} + \nu\right]^{1/\varphi}} \tag{16}$$

A permanent global warming (higher $\overline{\omega}$) lowers growth (γ). The effect on the fraction of final output invested to replenish renewable capital, i_y^g is nonlinear. It depends on the elasticity of K_t^g/K_t^p with respect to the climate shock ($\overline{\omega}$).

4. Carbon tax in a decentralized economy

We now turn to a competitive decentralized economy with households, firms and government and design a carbon tax which will replicate the Pareto optimal growth (**??**) of the command economy. Competitive firms produce final goods using the production function (2). The representative household owns the nonrenewable capital, accumulate it and rent it at a competitive rental price (r_t) every period to a firm for final goods production. Since nonrenewable capital is owned by the household, it will be referred as private capital hereafter. The profit (Π_t) from running the firm is rebated to the household as lump sum transfer. While producing final goods, the private sector does not internalize the damage caused to green capital based on (5).

The government imposes a carbon tax (τ_t) on household's rental income in a Pigovian fashion to correct for the externality and uses the tax proceeds to finance investment in renewable capital which we call green investment hereafter. Any shortfall or surplus from financing such green investment is rebated to the household as a lump sum transfer (T_t). The government budget constraint is thus:

$$\tau_t r_t K_t^p = i_{yt}^g Y_t + T_t \tag{17}$$

where the green investment ratio $\{i_{yt}^g\}$ satisfies (6).

The household takes the stock of renewable capital $\{K_t^g\}$ as well as the sequences $\{\tau_t\}$, $\{T_t\}$, $\{\Pi_t\}$ and $\{r_t\}$ as parametrically given, and maximizes (8) subject to the following flow budget constraint and the private investment technology (3):

$$C_t + I_t^p = (1-\tau_t) r_t K_t^p + \Pi_t + T_t \tag{18}$$

The Euler equation facing the household is:

$$\frac{C_{t+1}}{C_t} = \beta \left[(1-\tau_{t+1}) r_{t+1} + 1 - \delta_p \right] \tag{19}$$

The representative firm takes the stock of renewable capital (K_t^g) as given and chooses the private capital to maximize the following profit function:

$$\Pi_t = \underset{K_t^p}{Max} A \left[(1-\nu) K_t^{p^{\varphi}} + \nu K_t^{g^{\varphi}} \right]^{1/\varphi} - r_t K_t^p \tag{20}$$

The competitive rental price of capital (r_t) equals the marginal product of private capital which means

$$r_t = \Theta \left(\frac{K_t^g}{K_t^p} \right) \tag{21}$$

4.1. Optimal carbon tax

The time path of carbon tax to finance the green investment is determined such that the private marginal benefit of investing in non-renewable capital exactly balances the social marginal benefit given by the social planner's Euler equation (14). Based on (21) and (15), the optimal carbon tax formula is:

$$\tau_t = \frac{\overline{\omega}}{\Theta\left(\frac{K_t^g}{K_t^p}\right)} \tag{22}$$

Plugging the efficient time path of K_t^g/K_t^p from the social planning problem, one can generate the time path of the carbon tax, τ_t. The carbon tax rate in (22) is constant along the balanced growth path because K_t^g/K_t^p is constant. The elasticity of the tax rate with respect to emission is given by

$$\frac{\partial \ln \tau_t}{\partial \ln \overline{\omega}} = 1 - \frac{\partial \ln \Theta\left(\frac{K_t^g}{K_t^p}\right)}{\partial \ln \frac{K_t^g}{K_t^p}} \frac{\partial \ln \frac{K_t^g}{K_t^p}}{\partial \ln \overline{\omega}}$$

For plausible parameter values, this elasticity is positive. A positive climate shock triggers a higher carbon tax. The simulation reported later confirms it.

Given the time path of Pareto efficient public investment rate (i_{yt}^g) from (16), the government then designs a time path of the optimal transfer per unit of private capital based on the government budget constraint (17).

$$\frac{T_t}{K_t^p} = \overline{\omega} - i_{yt}^g \frac{Y_t}{K_t^p} \tag{23}$$

5. Model simulations

We perform model simulations of our baseline production based emission model to assess the long run effects of climate shock on the aggregate economy. In order to carry out any quantitative exercise, we take a stand on setting the long run growth target for the UK economy. We set a baseline target growth rate for the UK economy of 2% at a zero emission rate. This target is in line with the long term annual average growth rate of UK real GDP over the period 1947-2018 from the St. Louis Federal Reserve database (FRED) which is found to be 2.47%. One may debate whether this is a reasonable target given that the UK economy, in recent years, has slowed down (1.47% in 2019). Since there are no reliable GDP growth rate forecasts for the UK, we take 2% as a reasonable growth target.

Regarding the choice of the value of the discount factor β opinions considerably differ. Prescott (1986) sets β equal to 0.96 for calibrating the US economy to annual data which means a 4% steady state real interest rate. This estimate is used in many calibration exercises of macro models. Given the assumption of a logarithmic utility function, which is also widely used in quantitative macroeconomics literature following Prescott (1996), a 2% growth rate together with β equal to 0.96 implies a social discount rate of 6%.[11] This social discount rate is too high in the context of climate change involving the future generation's welfare. Green Book (2018) suggests

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[11] For a mature economy on a balanced growth path, the so called accounting rate of interest is equal to the consumption rate of interest. The standard rule in social cost benefit literature is that along the balanced growth path, the social discount rate (ρ) is equal to growth rate (g) times the intertemporal elasticity of substitution in consumption (say, σ) plus the impatience rate $(1-\beta)$. See Bell (2003, Ch 10) for a discussion of this and other rules for ρ. Given our $g = 0.02$ and $\sigma = 1$ due to our logarithmic utility assumption, it implies that $\rho = 0.04$.

that the social discount rate is 3.5% based on a 2% growth rate and an implicit assumption of a logarithmic utility function. On the other hand, the Stern report (2007) takes a radical stand that the social discount rate is around 0.05%. We fix β equal to 0.98, which implies that the social discount rate is 4%. We also perform a sensitivity analysis in order to test how our quantitative analysis differs when β is changed in this neighborhood.

Following Prescott (1996), the depreciation rate of non-renewable capital, δ_p, is fixed at 0.1 which implies a 2.5% quarterly depreciation rate used in several studies. With all these parameter values, the TFP parameter, A, needs to be fixed at 0.127. The elasticity and share parameters are fixed at $\varphi = 0.5$ and $\nu = 0.5$, respectively. With these values, we obtain a long run annual growth target of 2% for the UK economy at $\overline{\omega} = 0$. At zero emissions ($\overline{\omega} = 0$), the ratio of consumption to GDP and man made investment to GDP are found to be 42.7% and 19.2%, respectively; while the green investment rate is 38.7%.

Figure 2 plots the steady state effect of global warming on the aggregate economy. Starting from a zero erosion in response to a higher $\overline{\omega}$, the carbon tax rate rises sharply from zero to a rate which induces firms to substitute non-renewable capital for renewable capital. Public investment rate in renewable capital required to replenish such renewable capital rises while the rate of private investment falls. The consumption rate of the current generation rises, which reflects a substitution effect of carbon tax encouraging the household to consume more and invest less in non-renewable capital. The green depreciation rises (δ_{gt}) because the rise in the ratio of renewable to non-renewable capital is not enough to lower the depreciation of renewable capital. However, the green depreciation rate remains close to zero. The lower growth reflects the deadweight loss imposed by emission. Although the consumption rate is higher, the negative growth effect depresses the steady state societal welfare.[12] Not surprisingly, the global warming inflicts a deadweight loss on the society which cannot be mitigated by a carbon tax.

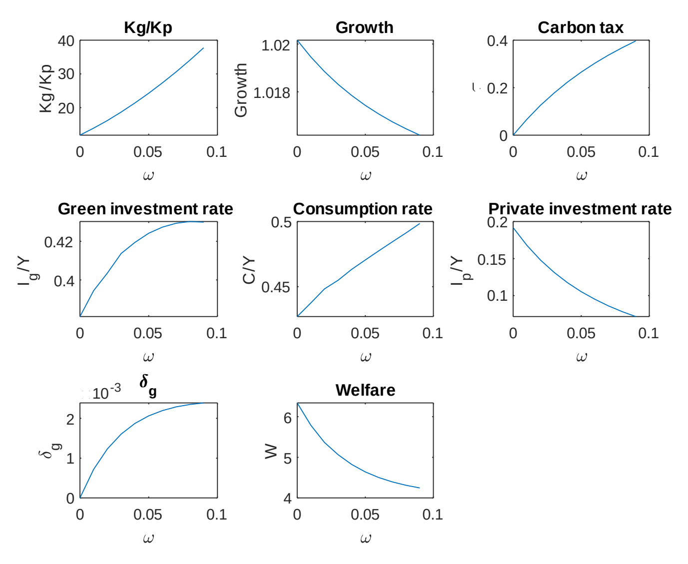

Figure 2: Effects of a permament global warming

Anticipating disparate opinions about the choice of the social discount rate, we perform a sensitivity analysis of the key variables by changing the social discount rate. Changing the discount factor β from 0.98 to 0.995 is equivalent to changing the social

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[12] The steady state welfare is computed as:

$$\frac{\ln Y_0 + \ln(C_0/Y_0)}{1-\beta} + \frac{\beta \ln G}{(1-\beta)^2}$$

discount rate from 4 to 2.5% given the same balanced growth rate of 2%. Table 1 presents the results of this sensitivity analysis. A lower social rate of time preference for future consumption raises the target growth rate from 2 to 3.58%. This increase in green investment takes place at the expense of a drastic reduction in man-made investment rate and lower societal consumption rate.

Table 1: Sensitivity of zero emission targets with respect to the social discount rate

β	0.98	0.985	0.99	0.995
γ	2.02%	2.54%	3.06%	3.58%
i_g/y	38.10%	47.9%	57.78%	67.63%
i_p/y	19.2%	20.03%	21.36%	10.68%
c/y	42.7%	32.03%	20.86%	21.69%

5.1. Sustainability and the Carbon Tax

Proponents of strong sustainability (e.g, Daly, 1997; Ayres, 2007) disallow substitution between renewable and nonrenewable capital while Solow (1974) and Nordhaus (1977) take a weak sustainability stand that some degree of substitution may be allowed. We measure the degree of sustainability by the elasticity of substitution between these two types of capital. The lower the elasticity of substitution, we come closer to the view of the strong sustainability group. Table 2 reports the sensitivity of the carbon tax to different degrees of substitution between man made and renewable capital. As the elasticity of substitution decreases, the carbon tax progressively rises. The rise is sharper when rate of emission ϖ is higher. If the technology does not permit easy substitution between these two types of capital, the private agents have to be charged for emission so that replenishment can be done with green investment. For a high rate of emission (such as ϖ =0.04), the carbon tax could range up to 31%.

Table 2: Sensitivity of Carbon Tax to the Elasticity of Substitution between green and man made capital

$\sigma \rightarrow$ $\varpi \downarrow$	3.33	1.43	1.00	0.17	0.02	0.01
0.01	6.31%	6.92%	7.19%	8.13%	8.40%	8.42%
0.02	11.92%	13.03%	13.57%	15.51%	16.10%	16.15%
0.03	16.95%	18.46%	19.25%	22.20%	23.30%	23.46%
0.04	21.28%	23.30%	24.31%	28.23%	31.06%	31.38%

5.2. Green bias and growth

How is growth impacted if we switch to a technology with a green bias? The green bias is measured by the technology parameter ν in the aggregate production function (2). A larger value of ν means a greater bias in the technology in favour of using renewable capital. For a Cobb Douglas production function with elasticity of substitution ($\sigma = 1$), the parameter ν turns out to be the share of renewable capital in aggregate output. Table 4 reports the sensitivity analysis of steady state growth, factor intensity and carbon tax in a scenario with zero emission. All other parameters are fixed at the same levels as in the baseline model. As the green bias increases long run growth unambiguously rises. This happens because the green factor intensity (K^g/K^p) rises which raises the marginal product of man made capital, $\Theta\left(K_t^g/K_t^p\right)$ in (11) and lowers the carbon tax in (22).

Table 3: Effect of green bias in the technology

ν	0.5	0.6	0.7	0.8
Growth	1.94%	3.12%	4.51%	6.19%
K^g/K^p	13.92	21.68	38.27	77.16
τ	6.66%	6.16%	5.67%	5.17%

6. Achieving net-zero emission target through an efficient pollution abatement technology.

There has long been a simple proposal in the policy parlance to address carbon emission using some form of a pollution abatement technology. There is a renewed interest in a carbon capture usage and storage (CCUS) technology to achieve the zero net emission target.[13] Such a CCUS technology could be expensive but can be financed by carbon tax revenue. We consider in our model the possibility of a similar

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[13] A CCS technology alone may not be adequate for pollution abatement. There are also other ways of abating pollution. The Global Commission on the Economy and Climate in their technical report suggests several pathways for this, which include: (i) more compact urban form with greater use of public transport, (ii) improving agricultural productivity, (iii) removal of fossil fuel subsidies, (iv) transition from coal, (v) phasing out short lived climate pollutants such as black carbon, methane, HFCs, (vi) emissions from oil and gas, (vii) reduced food wastage called waste resource action programme (WRAP). See
https://newclimateeconomy.report/workingpapers/wp-content/uploads/sites/5/2016/04/NCE-technical-note-emission-reduction-potential_final.pdf

technology via public investment in pollution abatement.

Suppose in addition, to green investment (i^g_{yt}), a fraction of GDP (i^ω_{yt}) is spent on emissions abatement. Formally, we introduce a simple linear emissions abatement technology as follows:

$$\omega_t = \varpi - \varkappa i^\omega_{yt} \tag{24}$$

If there is no public investment in emission abatement, emission is simply ϖ. The higher the investment in emissions abatement, the lower the emissions via the abatement technology (24). The efficiency of the emissions abatement is captured by the parameter $\varkappa$ which is the marginal pollution abatement of i^ω_{yt}.

The policy authority sets the time path of abatement investment such that the net-zero carbon emission ($\omega_t = 0$) is achieved in the long run. This means that the abatement investment rate is:

$$i^\omega_{yt} = \frac{\varpi}{\varkappa} \tag{25}$$

We assume that $\varkappa > \varpi$ to keep the abatement investment rate bound above by unity.

The social planning problem (8) now changes to:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{26}$$

s.t.

$$C_t + I^p_t \leq (1 - i^g_{yt} - i^\omega_{yt})Y_t; \quad K^g_0,\ K^p_0 = \text{given} \tag{27}$$

and (1), (2), (3), (6), (9) and $i^g_{yt} < 1$.

As before, the economy instantly reaches the steady state without any transitional dynamics. The balanced growth equation (14) now nets out the abatement investment. It is given by:

$$1+\gamma=\beta\left[(1-\frac{\varpi}{\varkappa})\Theta\left(\frac{K_t^g}{K_t^p}\right)+1-\delta_p\right] \tag{28}$$

The static efficiency condition (10) is modified after including abatement investment (25) as follows:[14]

$$\Theta\left(\frac{K_t^g}{K_t^p}\right)-\frac{\delta_p}{1-(\frac{\varpi}{\varkappa})}=\Psi\left(\frac{K_t^g}{K_t^p}\right) \tag{29}$$

Since the net emission is targeted at zero, the steady state green investment ratio (16) changes to:

$$i_y^g=\frac{\gamma}{A\left[(1-\nu)(K_t^p/K_t^g)^{-\varphi}+\nu\right]^{1/\varphi}} \tag{30}$$

In a decentralized economy, the government budget constraint changes to

$$\tau_t r_t K_t^p=(i_{yt}^g+i_{yt}^\omega)Y_t+T_t \tag{31}$$

Any surplus or shortfall to finance green investment and abatement investment is rebated to the household as a lump sum transfer.

Proposition 2. *The optimal carbon tax (τ_t) is simply the rate of abatement investment $\frac{\varpi}{\varkappa}$.*

Proof. To see it note that the household's private investment decision is still guided by the Euler equation (19). The social planner's private capital accumulation is given

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[14] The appendix presents an outline of the key equations of this model.

by the following Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta \left[(1 - \frac{\varpi}{\varkappa}) \Theta \left(\frac{K_t^g}{K_t^p} \right) + 1 - \delta_p \right]$$

Given that the equilibrium rental price of capital is given by (21), it immediately follows that $\tau_t = \frac{\varpi}{\varkappa}$. ■

Greater efficiency of an abatement technology is an indirect subsidy to private capital as it lowers the carbon tax. This subsidy lowers the ratio of green to man made capital as summarized in the following proposition.

Proposition 3. *Green factor intensity is lower for a more efficient abatement technology. In other words,* $\frac{\partial K_t^g / K_t^p}{\partial \chi} < 0$.

Proof. From the static efficiency condition (29) note that

$$\frac{\partial K_t^g / K_t^p}{\partial \chi} = \frac{-\delta_p \varpi}{(\varkappa - \varpi)^2 \left[\Theta' \left(\frac{K_t^g}{K_t^p} \right) - \Psi' \left(\frac{K_t^g}{K_t^p} \right) \right]}$$

Since $\Theta' \left(\frac{K_t^g}{K_t^p} \right) > 0$ and $\Psi' \left(\frac{K_t^g}{K_t^p} \right) < 0$, it immediately follows that $\frac{\partial K_t^g / K_t^p}{\partial \chi} < 0$. ■

Figure 3 plots an illustrative simulation of the effects of a higher χ on the aggregate economy. The initial emission ϖ is fixed at 0.02. Other structural parameters are recalibrated from previous values in Figure 2 to keep the long run growth rate in the vicinity of 2%. All relevant endogenous steady state variables are plotted against various values of the abatement efficiency parameter χ starting from 0.2. For χ =0.2, the abatement investment rate is about 10% of GDP which can be drastically lowered to 3% when χ is raised to 0.65. This also means a lower carbon tax. The ratio of green to man made capital falls as abatement technology becomes more efficient.

The long run growth rises from 1.6% to 2.1%. Both green and private investment rate also rise. The overall effect on steady state welfare is positive.

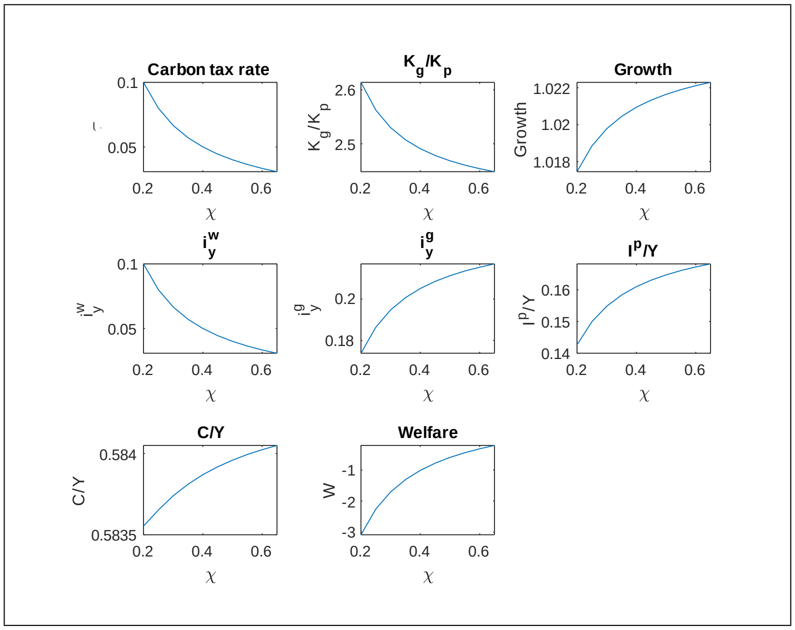

Figure 3: The effect of a more efficient pollution abatement technology

7. Conclusion

This paper addresses a topical issue: is sustainable growth possible with a shift to renewable resources? The problem is challenging because renewable resources use a massive amount of finite natural resources and the private producers do not necessarily internalize the emissions from fossil fuel intensive resources. Using the lens of a standard endogenous growth model, we show that a reasonable carbon tax and public investment in the form of augmenting the stock of renewable and pollution abatement can mitigate these problems and help us achieve the net-zero green target. However, to attain such goal, the extant production technology has to allow sufficient substitution of non-renewable by renewable capital. Currently, the substitutability of green and polluting capital is limited. This means that significant technological progress is needed to increase these substitution possibilities.

An important policy lesson learnt from our model environment is that just charging polluters will not be enough to reach the net-zero carbon target in 2050. One also needs a more efficient pollution abatement technology to achieve this target. Greater efficiency of a pollution abatement technology can deliver greener environment and higher sustainable growth and societal welfare. The adverse effect of emissions on growth can be reversed by an emissions abatement technology in the form of carbon capture and usage solutions such as forestation, and carbon capture and storage. In addition, this alternative technology should be supplemented by more green investment. Our model also demonstrates that switching to production technology with a greener bias can have positive effects on growth.

Our model can be extended in several directions. Since renewable capital uses large amounts of scarce natural resources, it is still a challenge how to efficiently use these natural resources to attain sustainable growth. Recycling resources may be a way to go which we do not model here. We also abstract from transitional dynamics in our model because of our central focus on long run growth with a net-zero emissions target. Introducing pollution distaste and adverse health effects of emission can give rise to rich transitional dynamics. These extensions can enrich the model, but are unlikely to alter the key results.

A. Appendix

The present value Lagrangian is given by:

$$\begin{aligned} L^p = & \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t \left[(1 - i_{yt}^g) A K_t + (1 - \delta_p) K_t^p - C_t - K_{t+1}^p \right] \\ & + \sum_{t=0}^{\infty} \mu_t \left[K_t^g + i_{yt}^g A K_t - \omega K_t^p - K_{t+1}^g \right] \end{aligned} \tag{A.1}$$

where $\{\lambda_t\}$ and $\{\mu_t\}$ are the Lagrange multipliers. The first order conditions are:

$$C_t : \ \beta^t / C_t \ - \lambda_t = 0 \tag{A.2}$$

$$K_{t+1}^p : \ \ - \lambda_t + \lambda_{t+1} \left\{ (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p \right\} - \mu_{t+1} \omega + \mu_{t+1} A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^p} = 0 \tag{A.3}$$

$$K_{t+1}^g : \ \lambda_{t+1} (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} - \mu_t + \mu_{t+1} \left\{ 1 + A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^g} \right\} = 0 \tag{A.4}$$

$$i_{yt}^g : \ \ - \lambda_t + \mu_t = 0 \tag{A.5}$$

Eq (A.5) is the foundation of the crucial static efficiency condition that equates the marginal distortion from the tax rate to the marginal benefit of the tax to finance green capital. Plugging (A.5) into (A.3) and using (A.2), we get:

$$\frac{C_{t+1}}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p - \bar{\omega} \right] \tag{A.6}$$

Likewise, plugging (A.5) into (A.4) and using (A.2), we get:

$$\frac{C_{t+1}}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} + 1 \right] \tag{A.7}$$

Equating (A.6) to (A.7), one obtains the static efficiency condition (9).

To get the optimal carbon tax formula (22), equate the right hand sides of (A.6) and (A.7).

A.1. Model with pollution abatement

The present value Lagrangian is given by:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \overline{\lambda}_t \left[(1 - i^g_{yt} - \frac{\varpi}{\varkappa}) A K_t + (1-\delta_p) K^p_t - C_t - K^p_{t+1} \right] \tag{A.8}$$
$$+ \sum_{t=0}^{\infty} \overline{\mu}_t \left[K^g_t + i^g_{yt} A K_t - K^g_{t+1} \right]$$

where $\{\overline{\lambda}_t\}$ and $\{\overline{\mu}_t\}$ are the Lagrange multipliers. The first order conditions are:

$$C_t : \ \beta^t / C_t \ - \overline{\lambda}_t = 0 \tag{A.9}$$

$$K^p_{t+1} : -\overline{\lambda}_t + \overline{\lambda}_{t+1} \left\{ (1 - i^g_{yt+1} - \frac{\varpi}{\varkappa}) A \frac{\partial K_{t+1}}{\partial K^p_{t+1}} + 1 - \delta_p \right\} + \overline{\mu}_{t+1} \left\{ A i^g_{yt+1} \frac{\partial K_{t+1}}{\partial K^p_{t+1}} \right\} = 0 \tag{A.10}$$

$$K^g_{t+1} : \overline{\lambda}_{t+1} (1 - i^g_{yt+1} - \frac{\varpi}{\varkappa}) A \frac{\partial K_{t+1}}{\partial K^g_{t+1}} - \overline{\mu}_t + \overline{\mu}_{t+1} \left\{ 1 + A i^g_{yt+1} \frac{\partial K_{t+1}}{\partial K^g_{t+1}} \right\} = 0 \tag{A.11}$$

$$i^g_{yt} : \ \ - \overline{\lambda}_t + \overline{\mu}_t = 0 \tag{A.12}$$

Combining (A.9), (A.10), (A.11) and (A.12), one gets

$$\frac{C_{t+1}}{C_t} = \beta \left[(1 - \frac{\varpi}{\varkappa}) \Theta \left(\frac{K^g_t}{K^p_t} \right) + 1 - \delta_p \right]$$
$$\frac{C_{t+1}}{C_t} = \beta \left[1 + (1 - \frac{\varpi}{\varkappa}) \Psi \left(\frac{K^g_t}{K^p_t} \right) \right]$$

which together yield the static efficiency condition (29).

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