Maximum Penalty and the Optimal Timing of Audits

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ABSTRACT

We study a setting where both a regulator and a firm may detect and stop bad projects through auditing. We show that the firm's auditing incentive under limited liability varies with the timing of audits — whether the regulator audits before or after the firm conducts its own audit. Moreover, the optimal timing of audits depends on the size of penalty that the regulator can impose when it detects a bad project.

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1 Introduction

Feess and Schumacher (2006) (F&S thereafter) study a model where both a regulator and a firm may detect and stop bad projects through auditing. They find that full auditing by the regulator may be socially suboptimal even with zero auditing cost. The reason is that full auditing by the regulator may crowd out the firm's own incentive to audit under limited liability. We show that the firm's auditing incentive under limited liability depends on the timing of audits — whether the regulator audits before or after the firm conducts its own audit. It is optimal for the regulator to audit after the firm's audit, if the regulator can impose a sufficiently large penalty on the firm when it detects a bad project; otherwise, it is optimal for the regulator to audit before the firm's audit.

2 Model

As in F&S, we consider a firm with equity E that owns a project that is good with probability $0 < q < 1$ and bad with probability $1 - q$. Investment costs are normalized to zero. Both kinds of projects yield a certain return R, but a bad project causes harm $D = 1$. The project is socially valuable *ex ante* but bad projects should not be carried out, hence $1 - q < R < 1$. Moreover, both the firm and the regulator can detect bad projects with some probability $0 < p < 1$ through auditing. To focus on the different timings of auditing, we assume that the auditing cost, $c > 0$, is the same for the two parties[1]. Both parties may audit only with some probability (or intensity). The firm's auditing effort is neither observable nor verifiable.

We consider the following two alternative timings that differ at Step 2 — whether the regulator audits before (*Ex Ante* auditing) or after (*Ex Post* auditing) the firm conducts its own audit.

Step 1. The regulator announces the timing and the probability (α) of its auditing, the penalty or fine (F) if a bad project is detected, and the liability if harm occurs.

Step 2. There are two stages at this step:

With *Ex Ante* auditing (as in F&S), the regulator audits with probability α in Stage 1. If it detects a bad project, it imposes the penalty on the firm and the game ends; if not,

[1] F&S assume the regulator's cost of auditing is zero.

the project continues to Stage 2. In Stage 2, the firm audits with probability β and stops bad projects.

With *Ex Post* auditing, the firm audits with probability β and stops bad projects in Stage 1. If the project continues to Stage 2, the regulator audits with probability α. If the regulator detects a bad project, it imposes the penalty on the firm and the game ends.

Step 3. If the project continues and harm occurs, the liability is imposed on the firm.

We assume

$$p(1-q)(1-p)(1-R) > c \tag{1}$$

so that it is socially efficient for both parties to audit with probability of 1.

It is well known that the firm would make the socially efficient auditing decision if it were strictly liable for any harm caused by the project. However, in reality, firms are often protected by limited liabilities. We assume that $E+R<1$, and the maximum the regulator can do is to leave the firm with nothing if harm occurs. Moreover, we assume that there is certain upper limit, $\overline{F}$, on the maximum fine the regulator can impose if it detects a bad project through auditing.

3 *Ex Ante* Auditing

Suppose the regulator audits with probability α in Stage 1 and the project continues to Stage 2. In Stage 2, the probability that the project is good is

$$q_\alpha = \frac{q}{q+(1-q)(1-\alpha)+(1-q)\alpha(1-p)} > q$$

With limited liability, the firm's expected profit is

$$\pi^A = q_\alpha R - (1-q_\alpha)(1-p)E - c$$

if it chooses to audit, and is

$$\pi^{NA} = q_\alpha R - (1-q_\alpha)E$$

if it chooses not to audit. Therefore, the firm audits if $\pi^A \geqslant \pi^{NA}$ or $c \leqslant p(1-q)E$. Notice that the size of penalty, F, has no effect on the firm's auditing decision and therefore does not affect the social welfare.

Lemma 1 *(Lemma 1 in F&S) Three cases exist regarding the firm's auditing decision: case 1. if $E < c/[p(1-q)]$, the firm invests without auditing for all α; case 2. if $E \geqslant$*

$c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, *the firm audits for all* α; *case 3. if* $c/[p(1-q)] \leqslant E < c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, *the firm audits if and only if* $\alpha \leqslant \overline{\alpha} = [pE(1-q) - c]/[p(pE(1-q) - c(1-q))]$. *The firm's auditing incentive is increasing in* E, *decreasing in* α *and* c, *and independent of* R. *The firm audits with less than efficient intensity.*

Proof. See F&S. ■

In the first two cases, because that the firm's loss in case of a bad project – its equity E – is either so high or so low, the firm either audit or does not audit regardless of the regulator's auditing intensity. When its equity is in-between, however, the firm audits only if the regulator's auditing intensity is sufficiently low. The reason is that the firm is more likely to have a good project in Stage 2 as the regulator's auditing intensity in stage 1 increases. Consequently, the firm's auditing incentive decreases as the regulator's auditing intensity increases.

Given the firm's behavior, it is optimal for the regulator to audit with probability of 1 in cases 1 and 2 because the firm's behavior is independent of α. In case 3, however, the regulator has two *choices*: $\alpha = 1$ and $\alpha = \overline{\alpha}$.

With $\alpha = 1$, the expected social welfare is

$$AW_1 = qR - (1-q)(1-p)(1-R) - c.$$

With $\alpha = \overline{\alpha}$, the expected social welfare is

$$AW_2 = qR - (1-q)(1-\overline{\alpha}p)(1-p)(1-R) - [q + (1-q)(1-\overline{\alpha}p) + \overline{\alpha}]c.$$

Since $AW_2 > AW_1$ for any $\overline{\alpha} > 0$ under condition (1), it is optimal for the regulator to audit with probability $\alpha = \overline{\alpha}$ in Case 3. Then, Proposition 1 describes the equilibrium outcome with *Ex Ante* auditing.

Proposition 1 *Three regions exist: region 1. if* $E < c/[p(1-q)]$, *the regulator always audits (i.e.,* $\alpha = 1$*) and the firm does not audit; regions 2. if* $E \geqslant c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, *both the firm and the regulator audit with probability of 1; region 3. if* $c/[p(1-q)] \leqslant E < c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, *the firm audits with* $\beta = 1$ *and the regulator audits with* $\overline{\alpha} < 1$.

In summary, the regulator audits with full intensity if the limited liability problem is either very constraining (region 1) or not constraining (region 2). Otherwise, the regulator audits with $\overline{\alpha} < 1$ which motivates the firm to audit with full intensity.

4 *Ex Post* Auditing

Suppose the firm audits with probability β in Stage 1 and the project continues to Stage 2. In Stage 2, the probability that the project is good is

$$q_\beta = \frac{q}{q + (1-q)(1-\beta) + (1-q)\beta(1-p)} > q.$$

The expected welfare is

$$PW^A = q_\beta R - (1-q_\beta)(1-p)(1-R) - c$$

if the regulator audits, and is

$$PW^{NA} = q_\beta R - (1-q_\beta)(1-R)$$

if the regulator does not audit. Therefore, it is socially efficient for the regulator to audit for any $0 < q_\beta < 1$ given condition (1).

In stage 1, with limited liability, the firm's expected profit is

$$\pi^A = qR - (1-q)(1-p)[\alpha pF + (1-\alpha p)E] - c$$

if it chooses to audit, and is

$$\pi^{NA} = qR - (1-q)[\alpha pF + (1-\alpha p)E)]$$

if it chooses not to audit. Therefore, the firm audits if $\pi^A \geqslant \pi^{NA}$ or $c \leqslant p(1-q)(\alpha pF + (1-\alpha p)E)$. Depending on the firm's equity E and the size of penalty F, multiple cases exist regarding the firm's auditing decision.

Lemma 2 *For $0 \leqslant F < E$, there exist three cases: case 1. if $E < c/[p(1-q)]$, the firm does not audit for all α; case 2. If $E \geqslant [c - p^2(1-q)F]/[p(1-p)(1-q)]$, the firm audits for all α; case 3. if $c/[p(1-q)] \leqslant E < [c-p^2(1-q)F]/[p(1-p)(1-q)]$, the firm audits if and only if $\alpha \leqslant \widetilde{\alpha} = [pE(1-q) - c]/[p^2(1-q)(E-F)]$. Moreover, the lower bound of E for case 3 decreases as F increases. For $F = E$, there exist only two cases: case 1. If $E < c/[p(1-q)]$, the firm does not audit for all α; case 2. If $E \geqslant c/[p(1-q)]$, the firm audits for all α. The firm's auditing incentive increases in F and E, and decreases in α and c. The firm makes the socially efficient auditing decision when $F = F^* = (1-\alpha p)(1-R-E)/(\alpha p)$.*

Proof. Define $\Phi \equiv \pi^A - \pi^{NA} = p(1-q)(\alpha pF + (1-\alpha p)E) - c$. Substituting $F = E$ into Φ provides $\Phi \geqslant (\leqslant) 0$ if $E \geqslant (\leqslant) c/[p(1-q)]$. For $0 \leqslant F < E$, substituting $\alpha = 0$

into Φ provides $\Phi > 0$ if $E > c/[p(1-q)]$, and substituting $\alpha = 1$ into Φ provides $\Phi > 0$ if $E > [c - p^2(1-q)F]/[p(1-p)(1-q)]$. Moreover, $\partial\Phi/\partial E = p(1-q)(1-\alpha p) > 0$ and $\partial\Phi/\partial\alpha = p^2(1-q)(F-E) \leqslant 0$. Therefore, the above three cases must exist with $\widetilde{\alpha}$ solved from $\Phi = p(1-q)(\widetilde{\alpha}pF + (1-\widetilde{\alpha}p)E) - c = 0$. Since $\partial\Phi/\partial F = \alpha p^2(1-q) > 0$ and $\partial\Phi/\partial E > 0$, the threshold E for case 3 decreases in F and converges to $c/p(1-q)$ as F converges to E. Finally, the firm audits for $c < p(1-q)(\alpha pF + (1-\alpha p)E)$, while it is socially efficient to audit for $c < p(1-q)(1-\alpha p)(1-R)$. Therefore, the firm makes the socially efficient auditing decision when $\alpha pF + (1-\alpha p)E = (1-\alpha p)(1-R)$ or $F = F^* = (1-\alpha p)(1-R-E)/(\alpha p)$. Given that $0 < F < E$, F^* exists for $(1-\alpha p)(1-R) < E < 1-R$. ■

A comparison between Lemmas 1&2 shows that the audit by the regulator affects the firm's expected profit (and therefore its auditing decision) differently with different timings. In particular, with *Ex Post* auditing, the regulator can influence the firm's auditing incentive through the size of penalty, and the firm may make the socially efficient auditing decision.

Given that double auditing is socially efficient, it is optimal for the regulator to set the penalty $F = \overline{F}$, as the firm's auditing incentive increases in F.

Moreover, the regulator will audit with probability of 1 either when $\overline{F} = E$ or in cases 1 and 2 when $0 \leqslant \overline{F} < E$, because the firm's auditing decision is independent of α in these cases. However, in case 3 when $0 \leqslant \overline{F} < E$, the regulator has two *choices*: $\alpha = 1$ and $\alpha = \widetilde{\alpha}$.

With $\alpha = 1$, the expected social welfare is

$$PW_1 = qR - (1-q)(1-p)(1-R) - c.$$

With $\alpha = \widetilde{\alpha}$, the expected social welfare is

$$PW_2 = qR - (1-q)(1-p)(1-\widetilde{\alpha}p)(1-R) - c - [q + (1-q)(1-p)]\widetilde{\alpha}c.$$

Notice that $PW_2 > PW_1$ for any $\widetilde{\alpha} > 0$ under condition (1). Therefore, the regulator audits with probability $\alpha = \widetilde{\alpha}$ if $c/p(1-q) \leqslant E < [c - p^2(1-q)\overline{F}]/[p(1-p)(1-q)]$ and $0 \leqslant \overline{F} < E$.

Proposition 2 *Three regions exist when $0 \leqslant \overline{F} < E$: region 1. if $E < c/[p(1-q)]$, the regulator always audits (i.e., $\alpha = 1$) but the firm does not audit; regions 2. if $E \geqslant [c - p^2(1-q)\overline{F}]/[p(1-p)(1-q)]$, both the firm and the regulator audit with probability of 1; region 3. if $c/[p(1-q)] \leqslant E < [c - p^2(1-q)\overline{F}]/[p(1-p)(1-q)]$, the firm audits with $\beta = 1$ and the regulator audits with $\alpha \leqslant \widetilde{\alpha} = [pE(1-q) - c]/[p^2(1-q)(E - \overline{F})]$.*

Two regions exist when $\overline{F} = E$: region 1. if $E < c/[p(1-q)]$, the regulator always audits (i.e., $\alpha = 1$) but the firm does not audit; region 2. if $E \geqslant c/[p(1-q)]$, both the firm and the regulator audit with probability of 1.

5 The Optimal Timing of Audits

Now we investigate the optimal timing of audits by comparing the equilibrium outcomes. As shown in Figure 1, for $E < c/[p(1-q)]$ or $E \geqslant c/[p(1-p)(1-q)]$, either *Ex Ante* or *Ex Post* auditing achieves the same outcome. That is, the regulator audits with $\alpha = 1$ and the firm does not audit when $E < c/[p(1-q)]$, and both the firm and the regulator audit with probability of 1 when $E \geqslant c/[p(1-p)(1-q)]$. For $c/[p(1-q)] \leqslant E < c/[p(1-p)(1-q)]$, the firm audits with probability of 1 regardless of the timing of auditing; however, the regulator's auditing intensity differs with different timings of auditing. Since $AW_2 = PW_2$ for $\overline{\alpha} = \widetilde{\alpha}$, $\partial AW_2/\partial\overline{\alpha} > 0$, and $\partial PW_2/\partial\widetilde{\alpha} > 0$, the timing of auditing that maximizes social welfare is the one where the regulator audits with more intensity at equilibrium.

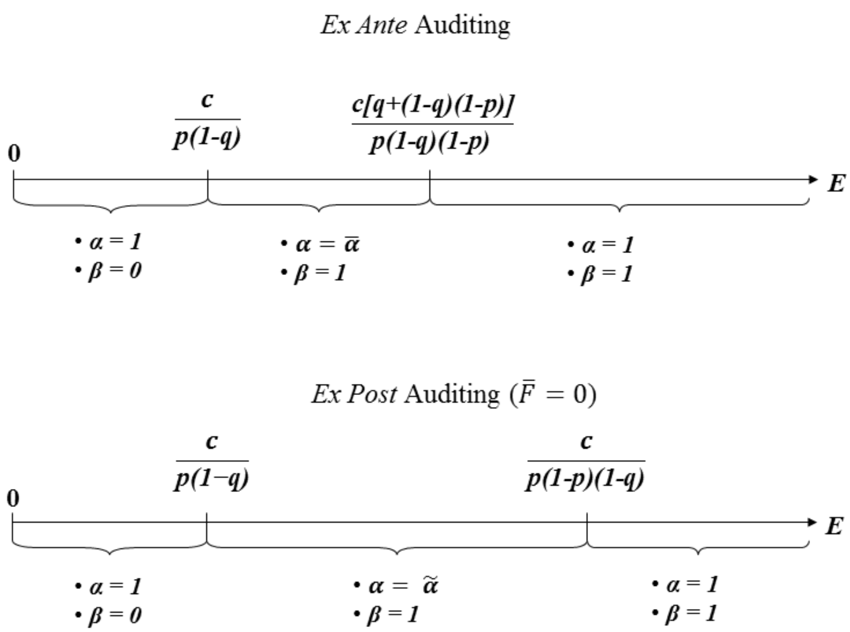

Figure 1. Equilibrium Outcomes with Different Timings of Auditing

Proposition 3 *For $c/p(1-q) < E < c/p(1-p)(1-q)$, there exists some $x(E)$ such that Ex Post Auditing is optimal if $\overline{F} \geqslant x(E)$ but Ex Ante Auditing is optimal if $\overline{F} < x(E)$.*

Proof. With *Ex Ante* auditing, as shown in Proposition 1, the regulator audits with probability $\overline{\alpha} < 1$ for $c/[p(1-q)] \leqslant E < c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, and with probability of 1 for $E \geqslant c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$. With *Ex Post* auditing, when $\overline{F} = E$, the regulator audits with probability of 1 for $E \geqslant c/[p(1-q)]$. Therefore, when $\overline{F} = E$, $PW_2(E) > AW_2(E)$ for $c/[p(1-q)] \leqslant E < c[q+(1-q)(1-p)]/[p(1-q)(1-p)]$, and $PW_2(E) = AW_2(E)$ for $c[q+(1-q)(1-p)]/[p(1-q)(1-p)] \leqslant E \leqslant c/[p(1-p)(1-q)]$. When $\overline{F} = 0$, under *Ex Post* auditing, the regulator audits with $\widetilde{\alpha} = [pE(1-q)-c]/[p^2(1-q)E] < \overline{\alpha}$ for $c/[p(1-q)] < E < c/[p(1-p)(1-q)]$. Therefore, when $\overline{F} = 0$, $PW_2(E) < AW_2(E)$ for $c/[p(1-q)] < E < c/[p(1-p)(1-q)]$. Since $\partial PW_2(E)/\partial\overline{F} = [\partial PW_2(E)/\partial\overline{\alpha}][\partial\overline{\alpha}/\partial\overline{F}] = [\partial PW_2(E)/\partial\overline{\alpha}][pE(1-q)-c]/[p^2(1-q)(E-\overline{F})^2] > 0$ for $E > c/[p(1-q)]$, there exists some $x(E)$ such that $PW_2(E) \geqslant AW_2(E)$ if $\overline{F} \geqslant x(E)$ and $PW_2(E) < AW_2(E)$ if $\overline{F} < x(E)$. ■

6 Conclusion

We study a setting where both a regulator and a firm may detect and stop bad projects through auditing. We show that the firm's auditing incentive under limited liability depends on the timing of audit — whether the regulator audits before or after the firm conducts its own audit. With *Ex Post* auditing, the firm's auditing incentive is affected by the size of penalty when a bad project is detected by the regulator; and the firm may make socially efficient auditing decisions if the penalty is sufficiently large. Consequently, *Ex Post* auditing is optimal if the regulator can impose a sufficiently large penalty. With *Ex Ante* auditing, the firm's auditing incentive doesn't depend on the size of penalty when the regulator detects a bad project, and the firm always audits with less than efficient intensity. However, *Ex Ante* Auditing is optimal when the regulator cannot impose a sufficiently large penalty.

References

[1] Feess, Eberhard, Schumacher, Christoph, 2006. Why costless auditing may reduce social welfare. *Economics Letters* 90, 407 - 411