Inequality, Growth, and Congestion Externalities

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Abstract

We describe, numerically simulate and empirically evaluate the aggregate and distributional properties of an endogenous growth model with an infrastructure externality which is subject to relative congestion. We show that the congested externality induces higher growth, greater inequality, labor/leisure trade-off ambiguities and an ineffective capital income tax for the government to achieve long-term redistribution goals. We demonstrate the economic implications of congestions in production and consumption externalities on the public to private capital ratio, growth and income distribution. Finally, we discuss alternative tax options for promoting inclusive growth.

Keywords: Congestion, externalities, growth, inequality, public capital.
JEL Classification: O1, O2, O3.

1. Introduction

This study evaluates the relationship between inequality and growth under circumstances where growth arises due to congestible infrastructure provision[1]. Accordingly, we seek to provide an intersection between three concurrent strands of literature. The first relates to the highly scrutinized, but as yet inconclusive, nexus between inequality and growth; which dates back to the early studies of Kuznets (1955) and Kaldor (1957). The second strand relates to the feedback between economic growth and infrastructure provision, which has its origins in the inquiry of Arrow and Kurz (1970),

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[1] Calderon and Serven (2014) highlights the fact that not all infrastructure is publicly provided, hence the non-interchangeability of the terms infrastructure and public capital in strict terms. However, in this paper we follow the traditional literature by using both terms to imply the same thing.

and has been empirically evaluated since Aschauer (1989). That literature is in general unified around the idea that infrastructure supports economic growth through the provision of productivity-enhancing externalities to private capital[2]. The third relationship involves the more nascent and relatively less well explored link between infrastructure and inequality as described for instance in Chatterjee and Turnovsky (2012).

The ambiguities arising from the first relationship are largely connected to the observation by Kuznets (1955) that the distributional consequences of growth are inextricably woven to the underlying source of such growth. In this regard, it is imperative to understand the unique imprint on income inequality that arises due to infrastructure provisioning in particular because 1) there is a subsisting massive global infrastructural deficit 2) the plugging of this deficit is likely to appropriate a significant fraction of most global economies' GDPs for at least the next 50 years and hence 3) the dynamics of growth and income inequality will therefore likely be influenced by infrastructural provisioning for the foreseeable future.

To provide an empirical benchmark, Figure 1 plots the relationship between the public-to-private capital ratio and income inequality amongst five major industrialized economies utilizing data from the IMF and WIID repositories for the public-to-private capital ratio and income inequality respectively. As the corresponding graphs show, the 55-year relationship between both variables suggests that inequality declines as the public-to-private capital ratio expands. Hence, we consider a negative relationship between this ratio and inequality as an empirical regularity that demands attention in the context of the growth-inequality nexus.

We explore the mechanism behind the above stylized fact using a heterogenous agent dynamic general equilibrium model with both private and public capital in pro-

[2]Surveys summarizing the empirical evidence of public capital productivity include Straub (2008), Boom and Ligthart (2010) and Calderon and Serven (2014).

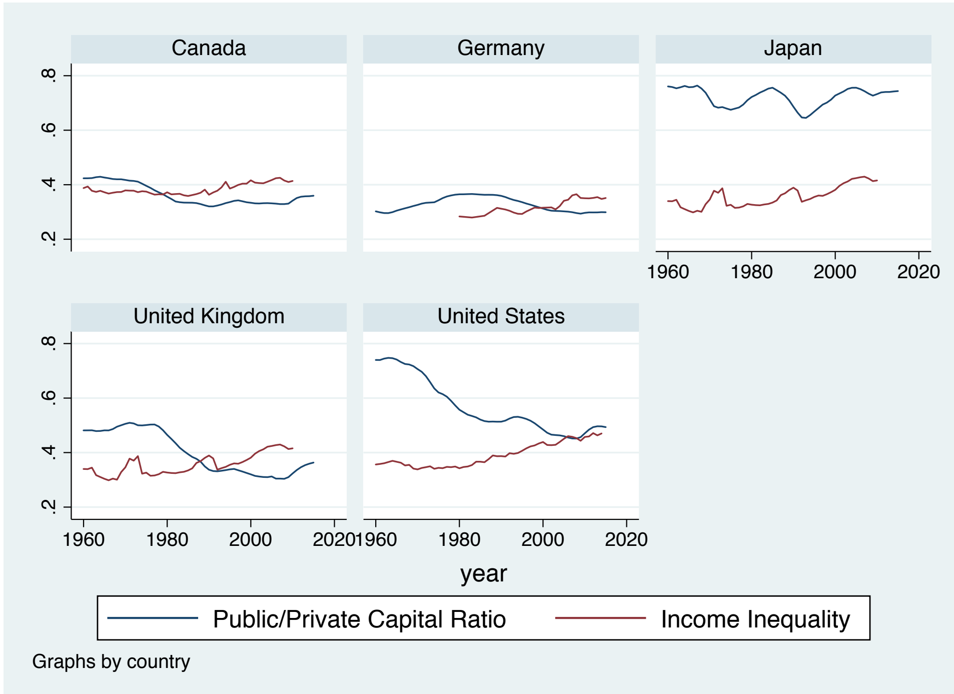


Figure 1: Public/Private Capital Ratio and Income Inequality. Income inequality is evaluated using the fraction of output accruing to the top decile. Source: IMF and WIID.

duction, and with special emphasis on the nature of the production externalities emanating from aggregate public and private capital. The resulting model has four key features. First, economic growth arises from the micro-founded utility maximizing objective of economic agents. Second, infrastructure is deployed in the form of an externality which supports private capital in production, and is financed by a flat tax, or a tax on consumption, labor, or capital income. Third, said infrastructure is subject to crowding in the form of relative congestion, and fourth, markets are perfect such that private agents can mitigate the productivity limitation associated with congestion through the acquisition of private capital.

The importance of explicitly accounting for congestion in the model arises due to the global infrastructural gap highlighted earlier, as indeed, empirical evidence suggests a widening global infrastructure deficit. This is described for instance in the ASCE 2017 report which indicates that the United States has to bridge a nearly $1.5 trillion infrastructure gap by 2025[3]. The equivalent figures for the OECD and Asia/Pacific region are \$6.3 trillion[4] and \$22.6 trillion[5]. In a similar vein, the 2018 estimates from the AfDB indicate that Africa requires on-going annual infrastructural expenditure to the tune of \$170 billion to meet on-going demand [6]. Not surprisingly therefore, incorporating congestion externalities into a dynamic equilibrium framework with infrastructure capital delivers modeling outcomes which bear stronger consistency with the data.

The resulting rivalry underlying the use of infrastructure, i.e. congestion, is noted by Barro and Sala-i-Martin (1992) as being a consistent feature of virtually all forms of publicly provided utilities. Research shows that the presence of congestion changes the behavior of macroeconomic agents, the resulting equilibrium outcomes, and the time-path between intertemporal transitions[7] . We can thusly infer that a role exists for the impact of congestion in the resulting infrastructure/inequality nexus. This connection has however remained crucially unaddressed in the extant literature.

Accordingly, the presence of congested public capital implies that private capital invariably assumes the dual roles of productive input and productivity supplement through its enhancement of access to infrastructure. This consequently leads to its over-accumulation, as highlighted in the literature (Fisher and Turnovsky, 1998; Eicher and Turnovsky, 2000) and consistent with Figure 1. As a result, the enhanced value of private capital is reflected in a higher equilibrium return to capital ownership through the rental rate. This leads to greater capital-induced income inequality alongside a faster equilibrium growth rate of output.

[3]https://www.infrastructurereportcard.org/

[4]https://www.oecd.org/env/cc/g20-climate/Technical-note-estimates-of-infrastructure-investment-needs.pdf

[5]https://www.adb.org/news/asia-infrastructure-needs-exceed-17-trillion-year-double-previous-estimates

[6]https://www.afdb.org/fileadmin/uploads/afdb/Documents/Publications/African_Economic_Outlook_2018_-_EN.pdf

[7] See for instance Fisher and Turnovsky (1998), Eicher and Turnovsky (2000) and Pintea and Turnovsky (2006).

Importantly, we find that the enhanced value of private capital weakens the redistributive efficacy of the capital income tax. Moreover, under conditions of sufficiently high relative congestion, taxing capital income to finance infrastructural upgrades becomes incapable of reversing the long-run trend of rising inequality, contrary to a key result in the literature as described in Chatterjee and Turnovsky (2012) and Klenert et.al (2018). An implication of the foregoing is that a government which engages in infrastructural provisioning as a means towards reducing income inequality, in the spirit of Andres et al. (2014), is likely to experience quite the opposite outcome in the presence of congestion.

We also extend the model to explore situations in which congestible externalities feature in consumption as proposed by Chatterjee and Ghosh (2011). Herein, we find that whereas agents would ordinarily substitute leisure for labor when pubic service increases in the case of the pure public good, if they perceive that accessing the benefits provided by the public good is conditional on how much private capital they possess, they are driven to demand a higher compensation for the rental of their capital and hence bid up its cost. This induces producers to substitute away from capital and towards the cheaper labor input in production. The dual impact of decreased supply of labor and greater demand thereof, raises the equilibrium wage which works to decrease income inequality. However, given the increased demand for capital arising from its greater value in utility, the equilibrium rental rate of capital is also driven up which nullifies the former effect.

Consequently, long-run equilibrium income and welfare inequality are seen to increase by higher margins for higher degrees of consumption-induced congestion externalities. Accordingly, while we document a higher level of equilibrium income inequality as being associated with economic growth for higher degrees of congestion, the structure of the underlying inequality varies depending on whether congestion is production or consumption-induced.

The rest of the paper is organized as follows: in Section 2 we develop the model, in Section 3 we describe the equilibrium of the system and expound on the role of congestion therein, in Section 4 we describe the steady state and in Section 5 we detail the transition dynamics of the economy. Section 6 derives and discusses the distributional properties of the economy while Sections 7 and 8 utilize numerical simulations to analyze the relationship between inequality and growth. Section 9 discusses the empirical fit of the model, Section 10 conducts a sensitivity analysis, and Section 11 concludes.

2. The Model

We construct a closed economy model where agents are heterogenous only in the dimension of the initial level of capital ownership. It is an endogenously growing economy where both private and public capital are accumulated and a positive externality enhances both the output production and utility[8].

2.1. Production

The economy is inhabited by unit mass of firms, each of which utilizes labor L_j, capital K_j, a Hicks-neutral technology A, and an appropriated externality X_j as inputs to produce output, Y using a Cobb-Douglas production technology

$$Y_j = AK_j^{\alpha}(L_j X_j)^{1-\alpha} \tag{1a}$$

We drop subscripts when referring to aggregate quantities. The composite externality X, is generated from the aggregated private capital K and public capital K_G, but the individual producer can harness the composite production externality accord-

[8]The model introduced here is closely related to the modeling approach of Caselli and Ventura (2000), Garcia-Penalosa and Turnovsky (2006), Fisher and Turnovsky (1998) and Chatterjee and Turnovsky (2012)'s extension of the Futagami, Morita and Shibata (2002) framework

ing to the producer's ownership of capital specified by the following relationship

$$X_j = K^{1-\varepsilon} K_G^{\varepsilon} \left(\frac{K_j}{K} \right)^R \tag{1b}$$

The externality which supports growth is assumed to be a composite of the aggregate state of knowledge K, and the availed public capital K_G, which is a hybrid of the externalities proposed in Romer (1986) and **?** the specific combination of which is indexed by the parameter ε.

We assume that the externality is rivalrous but non-excludable[9]. This implies that an individual producer can appropriate only a fraction of the externality depending on her/his capital relative to the aggregate capital. The following example, due to Eicher and Turnovsky (2000), motivates the argument for relative congestion;

> *In the context of a growth model... relative congestion specifies the level of services derived by an individual from the provision of a public good in terms of the usage of her/his individual capital stock relative to the aggregate capital stock. An example of this is the service provided by highway usage. Unless an individual drives her/his car, s/he derives no service from a publicly provided highway, and in general, the services s/he derives depends upon her/his own usage relative to that of others in the economy, as total usage contributes to congestion.*

In the above equation, R parameterizes the extent of congestion and varies between 0 and 1[10] . When $R = 0$, the externality is a pure public input; and when $R = 1$, the externality bears semblance to a private input since it's accessibility is fully proportional to relative private capital ownership. This modification in the production environment changes the dynamics of the economy substantially and yields predictions consistent with the observed relationships between growth and inequality.

[9]The aggregate and distributional consequences of assuming that some or all of the externality is subject to excludability as described in Ott and Turnovsky (2006), are easily incorporated into this framework by calibrating the flat tax of Section 2.2 in the form of a user fee. The details of this argument are discussed in Appendix E.

[10] Fisher and Turnovsky (1998), Eicher and Turnovsky (2000).

Substituting Eq.(1b) into Eq.(1a) then gives the full production specification perceived by agents;

$$Y_j = A[K_G^{\varepsilon} K^{1-(\varepsilon+R)} L_j]^{1-\alpha} K_j^{R(1-\alpha)+\alpha} \tag{1c}$$

Firms choose capital and labor to maximize profits, hence the first order condition with regard to capital yields the marginal physical product of capital for the jth firm

$$\frac{\partial Y_j}{\partial K_j} = A(R(1-\alpha)+\alpha)[K_G^{\varepsilon} K^{1-(\varepsilon+R)} L_j]^{1-\alpha} K_j^{R(1-\alpha)+\alpha-1} \tag{2a}$$

The presence of perfect capital markets and no information friction implies that all firms will choose the same quantity of capital and labor so that the subscripts may be dropped. The interest rate, which is exogenous to firms, is accordingly given by

$$r = A(R(1-\alpha)+\alpha)L^{1-\alpha} z^{\varepsilon(1-\alpha)} \tag{2b}$$

where $z = K_G/K$.

Similarly optimizing for units of labor utilized in production yields the marginal physical product of labor

$$\frac{\partial Y_j}{\partial L_j} = \frac{A(1-\alpha)[K_G^{\varepsilon} L]^{1-\alpha} K^{1-\varepsilon(1-\alpha)}}{L} = w \tag{3}$$

where w represents the wage rate. Defining $y \equiv Y/K$ i.e. the average output-to-capital ratio and $\omega \equiv w/K$, the above system can be represented as

$$y = AL^{1-\alpha} z^{\varepsilon(1-\alpha)} \tag{4}$$

$$r = (R(1-\alpha)+\alpha)y \tag{5}$$

$$\omega = \frac{(1-\alpha)y}{L} \tag{6}$$

2.2. Consumers

We assume that in each period the economy is populated by a unit mass of infinitely lived agents individually indexed by the subscript i, who are identical in every respect except for their initial endowment of capital $K_{i,0}$. Consumers receive utility from consumption of the final good, C_i and leisure l_i. We also however assume that the stock of public capital avails consumers a measure of utility, X_i which is also subject to relative congestion. Chatterjee and Ghosh (2011) argues that roads and highways provide utility to consumers who like driving. Similarly, altruistic parents can derive utility from sending their kids to public schools. Utility is also obtainable from public consumption goods such as parks, law and order, and defense[11].

Accordingly, the consumer maximizes the following utility function[12] ;

$$U_i = \int_0^\infty \frac{1}{\gamma}\big[C_i\big(l_i X_i^h\big)^\eta\big]^\gamma e^{-\beta t} dt \tag{7a}$$

Where h parametrizes the relative importance of the public good in utility and X_i assumes the form;

$$X_i = K_G \left(\frac{K_i}{K}\right)^{Rc} \tag{7b}$$

The formulation (7b) presupposes that agents can increase their utility from public capital by increasing their private capital. This is however subject to relative congestion, indexed by the parameter Rc with is constrained between $0 \leq Rc \leq 1$. With $Rc = 0$, the public externality is perceived as a pure public good while with $Rc = 1$, the consumption externality is similar to a private good.

The intertemporal elasticity of substitution is given by $e \equiv 1/(1-\gamma)$ while η indicates the elasticity of leisure in utility. Standard assumptions on the parameters of

[11]More insight on the rationale for consumption externalities, are provided in Chatterjee and Ghosh (2011).

[12]This formulation is standard in the literature on inequality and growth as in Garcia-Penalosa and Turnovsky (2006), Garcia-Penalosa and Turnovsky (2007), and Chatterjee and Turnovsky (2012).

the utility function restrict the value of γ to be less than unity in order to generate a positive intertemporal elasticity of substitution, while η is non-negative so that agent satisfaction is non-decreasing in leisure. The consumer observes a capital accumulation constraint given by

$$\dot{K}_i = (1 - t_k)rK_i + (1 - t_w)w(1 - l_i) - (1 + t_c)C_i - T \tag{8}$$

Where t_k, t_w and t_c represent the proportional tax rates on capital income, labor income and consumption while T is a lump-sum tax, assumed as a proportion τ, of income, all of which are levied by the government. Consumers take real wage rate w, and real return on private capital r as given since they are determined in the competitive factor markets; they also take all the taxes as given. The current value Hamiltonian may consequently be stated as follows

$$\Omega^a = \frac{\left[C_i(l_i X_i^h)^\eta\right]^\gamma}{\gamma} e^{-\beta t} + \lambda_i e^{-\beta t}[(1 - t_k)rK_i + (1 - t_w)w(1 - l_i) - (1 + t_c)C_i - T - \dot{K}_i] \tag{9}$$

Each agent chooses the level of consumption, leisure, and the rate of capital accumulation, $\dot{K}_i$ to maximize utility, resulting in the following first order conditions

$$\Omega_c^a; C_i^{\gamma-1}\left(l_i X_i^h\right)^{\eta\gamma} = \lambda_i\left(1 + t_c\right) \tag{10}$$

$$\Omega_l^a; \eta C_i^\gamma l_i^{\eta\gamma-1} X_i^{h\eta\gamma} = \lambda_i\left(1 - t_w\right)w \tag{11}$$

$$\Omega_K^a; \frac{h\eta R_c}{K_i} C_i^\gamma l_i^{\eta\gamma} X_i^{h\eta\gamma} + \left(1 - t_k\right) r\lambda_i = \beta\lambda_i - \dot{\lambda}_i \tag{12}$$

Where λ_i indicates the shadow-price of private wealth and k_i is the wealth of individual i relative to the mean. Eq.(10) equates the individual's marginal utility of consumption to the tax-adjusted marginal utility of wealth, while Eq.(11) equates the marginal utility of leisure to the tax-adjusted wage appropriately measured in terms of shadow price of wealth. Finally, Eq.(12) is the intertemporal efficiency condition which implies the equalization of returns to consumption and capital in equilibrium. The left side of this equation portrays the total effects of accumulating capital which is composed of two parts. The first part is the benefit an additional unit of capital to utility through appropriation of more from public consumption goods while the second part is the after-tax return to capital.

From Eq.s(10) and (11), we obtain the marginal rate of substitution between leisure and consumption as

$$MRS;\ \frac{C_i}{l_i} = \frac{(1-t_w)w}{(1+t_c)\eta} = \frac{(1-t_w)\omega K}{(1+t_c)\eta} = \Omega K \tag{13}$$

which is identical for all consumers. We normalize each agent's periodic time endowment to equal unity, so that summing labor supplied across all agents leads to the aggregate labor supply identity; $1 - l = L$. As Eq.(13) indicates, the equilibrium supply of labor is determined by the exogenous taxes on labor income and consumption as well as the elasticity of labor supply. Higher taxes on wages work in the same direction as taxes on consumption by inducing a substitution away from labor and towards leisure hence decreasing equilibrium labor supply and the consumption to leisure ratio for each agent.

The transversality condition is given by

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{14}$$

which imposes the condition that the present discounted value of wealth at the end of an agent's planning horizon must be zero.

2.3. Government

Public capital is assumed to evolve according to the following rule

$$\dot{K}_G = G = \theta Y;\ 0 < \theta < 1 \tag{15}$$

where G is the current period real increase in the net stock of public capital assumed for tractability as a fixed fraction of output. The government operates a balanced budget which is represented as

$$t_w wL + t_k rK + t_c C + T = G = \theta Y \tag{16a}$$

where the lump-sum tax T, is also a fraction of output. Expressed in terms of average output to capital ratio yields the following

$$t_w \omega L + t_k r + t_c \Omega l + \tau y = \theta y \tag{16b}$$

From Eq.(16b), a structural shock which changes the steady state value of θ will generate a transition process which will involve adjustments in z and l so that ω, r and Ω, all functions of the current values of z and l, will be time-varying when outside of steady-state. This implies that for fixed values of t_k, t_w and t_c, the lump-sum tax rate τ, will necessarily adjust to ensure θ remains constant.

3. Macroeconomic Equilibrium

Summing Eq.(8) across all agents and dividing by aggregate private capital yields the economy-wide capital growth rate

$$\frac{\dot{K}}{K} = (1 - t_k)r + (1 - t_w)\omega(1 - l) - (1 + t_c)\Omega l - \tau y \tag{17}$$

We rewrite this equilibrium condition with regard to Eq.'s(10)-(12) as follows;

$$C_i^{\gamma-1} l_i^{\eta\gamma} K_G^{h\eta\gamma} = \lambda_i (1 + t_c) \tag{10'}$$

$$\eta C_i^{\gamma} l_i^{\eta\gamma-1} K_G^{h\eta\gamma} = \lambda_i (1 - t_l) w \tag{11'}$$

$$(1 + t_c) \eta h R_c \frac{C_i}{K_i} + (1 - t_k) r = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{12'}$$

Taking the growth rate of Eq.(10), we obtain the following

$$(\gamma - 1) \frac{\dot{C}_i}{C_i} + \eta\gamma \frac{\dot{l}_i}{l_i} - h\eta\gamma \frac{\dot{K}_G}{K_G} = \frac{\dot{\lambda}_i}{\lambda_i}; \tag{18}$$

which, when combined with the MRS, i.e. Eq.(13), conditionally reproduces the result of Garcia-Penalosa and Turnovsky (2006) that the equilibrium is defined by all agents choosing the same consumption and leisure growth rates such that

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_j}{C_j} = \frac{\dot{C}}{C}; \tag{19a}$$

$$\frac{\dot{l}_i}{l_i} = \frac{\dot{l}_j}{l_j} = \frac{\dot{l}}{l}; \tag{19b}$$

$$l_i = \pi_i l; \text{ where } \sum \pi_i = 1; \tag{19c}$$

π_i is the proportion of total leisure in this economy associable with the ith individual. Note the non-uniqueness of $\dot{\lambda}_i/\lambda_i$ across agents from Eq(12') hence a confinement of the above result to a special case. We restrict our analysis to the case where all agents choose the same C/K ratio. Combining Eq.s(16), (18) and (19) we get the growth rate of private capital

$$\frac{\dot{K}}{K} = y \left[(1 + R(1 - \alpha) - \theta) - \frac{(1 - t_w)(1 - \alpha)}{(1 + t_c) \eta} \frac{l}{1 - l} \right]. \tag{20}$$

As long as output exceeds consumption, the growth rate of private capital is positive. Moreover, the long-run balanced growth path of this economy features identical growth rates for public and private capital, as represented below;

$$\frac{\dot{z}}{z} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = y\left[\frac{\theta}{z} - (1 + R(1-\alpha) - \theta) - \frac{(1-t_w)(1-\alpha)}{(1+t_c)\,\eta}\frac{l}{1-l}\right]. \tag{21}$$

Accordingly, short-run divergences from the balanced growth path arise whenever public and private capital grow at separate rates. In this environment, such divergences are triggered by fiscal expansions (or contractions). Where the source is an expansionary fiscal policy shock such as an increase in θ, persistent adjustments in private capital accumulation will occur until the private value prospects created by the increase in the public capital growth rate are fully internalized by producers. The pace of accumulation of private capital is in general enhanced by a higher average and marginal product of capital so that the differential between the growth rates of the public and private capital stocks falls faster with congestion, leading in general to a quicker transition; the effect of diminishing marginal returns is however seen to also set in at a much faster pace so that the overall impact of congestion yields a shorter time-path between steady-states. Similarly, the response of leisure in transition which, depending on the policy tool activated, could either increase or decrease relative to the new steady state, is also enhanced.

The dynamic path of labor is given by the ratio

$$\dot{l} = \frac{J}{H} \tag{22}$$

Where

$$J = (1-t_k)\left(R\left(1-\alpha\right)+\alpha\right)y + (1+t_c)\,h\eta R_c\frac{C}{K} + h\eta\gamma\frac{\dot{K}_G}{K_G} - \beta - (1-\gamma)\,\frac{\dot{K}}{K} - \varepsilon\left(1-\gamma\right)\left(1-\alpha\right)\frac{\dot{z}}{z}$$

$$H = \frac{1-\gamma\left(1+\eta\right)}{l} + \frac{\alpha\left(1-\gamma\right)}{1-l}$$

Hence Eq.s(21) and (22) summarize the dynamics of the economy and dictate the time-paths from one steady state to another.

4. Steady State

The existence of a steady state requires all macroeconomic aggregates to grow at the same rate, the realization of which in turn depends on the stability of the system. Assuming such stability exists, we define the following features therein;

- Private capital, public capital and consumption grow at the same rate and
- The growth rate of leisure ceases, given that leisure is bounded in the long run.

Together, these conditions imply that the stationary variables $\dot{z} = \dot{l} = 0$, and from Equations (10), (12) and (21) can be represented as follows

$$\tilde{\psi} \equiv \frac{1}{1-\gamma}\left[(1+t_c)\,\eta h R_c \frac{\tilde{C}}{\tilde{K}} + (1-t_k)\,r - \beta\right] = \frac{1-\gamma\,(1+h\eta)}{1-\gamma}\frac{\theta\tilde{y}}{\tilde{z}} \tag{23a}$$

$$\frac{\theta}{\tilde{z}} = (1+R(1-\alpha)-\theta) - \frac{(1-t_w)(1-\alpha)}{(1+t_c)\,\eta}\frac{\tilde{l}}{1-\tilde{l}} \tag{23b}$$

We use tildes to indicate steady-state values. Accordingly, $\tilde{\psi}$ represents the economy's long run/steady-state growth rate. From Eq.(17), the steady-state growth rate and transversality condition also imply the following;

$$\frac{\tilde{C}}{\tilde{K}} = \tilde{c} > \frac{(1-t_w)\tilde{\omega}(1-\tilde{l}) - \tau\tilde{y}}{1+t_c} \tag{24}$$

This implies that the absence of explosive equilibria constrains agents to consume an amount in excess of their labor income, which in turn imposes an upper boundary condition for labor and consequently growth in the system[13].

[13]Eq.(24) also implies that $r > \psi$. For greater detail on the derivation of Eq.(24) as well as more extensive coverage on stable equilibrium paths, see Turnovsky (2000), Garcia-Penalosa and Turnovsky (2006)

Through its impact on both the equilibrium labor/leisure choice and the value of capital, the system Eq.(23), results in a non-trivial feedback between congestion and the economy's steady state growth rate which we detail in the following proposition.

Proposition 1: *Congestion externalities in production induces an over-accumulation of private capital, which leads to a decrease the equilibrium ratio of public to private capital and a decrease in equilibrium labor demand; Congestion externalities in consumption also induces an over-accumulation of private capital, this however decreases the supply of private capital and lowers the relative cost of labor in production which leads to an increase in equilibrium labor demand.*

Equilibrium leisure and the public-to-private capital ratio are related to congestion through the system Eq.(23). We highlight in Appendix D that for plausible parameter values, $\partial z/\partial R < 0, \partial l/\partial R > 0, \partial z/\partial R_c < 0$, and $\partial l/\partial R_c < 0$, which implies that congestion decreases the steady state ratio of public-to-private capital while the impact on equilibrium leisure depends on the degree to which production and utility are differentially impacted. In particular, whereas higher congestion in general induces a substitution of capital for labor which leads to higher equilibrium leisure, if agents perceive the need to acquire more capital to access the utility content of public spending, they bid up the price of capital which leads producers to consequently increase their demand for the complementary labor input. Hence, congestion generates conflicting outcomes for equilibrium labor depending on its source.

The foregoing however depends on the relative importance of government services in utility as captured by the parameter h. From Eq.(23a), setting $h = 0$ expunges the impact of R_c in the system. Moreover, one can also show that $\partial l/\partial h > 0$ which implies that agents will have less incentive to work if they can freely take advantage of the utility-availing services accorded by public capital.

At this point, we describe a strategy to accommodate the differential effects that R

and Rc imply for equilibrium leisure in this framework, and the subsequent calibration exercise that we conduct. Our preference is for an approach which aligns with the empirical evidence on infrastructure, inequality and growth. In that regard, Figure F.6 in Appendix F provides the observed pattern between leisure and income inequality for the five economies listed in the introduction. The graphs demonstrate a largely positive co-relationship between both variables, hence alluding to a joint outcome of rising income inequality alongside a shrinking z ratio and increasing l, all consistent with the pattern generated by congestion externalities in production. While this does not rule out a historical (or larger prospective) role for Rc in the inequality and growth interplay, we will pay closer attention to the role of productive externalities in the system, and confine the discussion on externalities in utility to a special case.

5. Equilibrium Dynamics

To characterize the dynamic path of the economy, the system (21) and (22) is linearized around steady state, yielding the corresponding two-variable system

$$\begin{bmatrix} \dot{z} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} z(t) - \tilde{z} \\ l(t) - \tilde{l} \end{bmatrix} \tag{25}$$

Details of which are provided in Appendix B. The system is saddle-point stable if $a_{11}a_{22} - a_{12}a_{21} < 0$; we utilize numerical simulations to establish the satisfaction of this condition. The resulting dynamical system demonstrates the property of noticeably shorter transitions for higher degrees of relative congestion as discussed in Pintea and Turnovsky (2006). The distributional implication of this is that a higher degree congestion moderates the immediate short-run reaction of agents to changes in the public-to-private capital ratio, shortens distributional transition, and accelerates the pace of consolidation to a new steady state.

Because the system evolves slowly between the initial fiscal impulse and the eventual steady state, the time t realization of z, l, and the consumption to capital ratio c are described by the following pair of equations;

$$z(t) = \tilde{z} + [z(0) - \tilde{z}] \, e^{\mu t} \tag{26}$$

$$l(t) = \tilde{l} + \frac{a_{21}}{(\mu - a_{22})} [z(t) - \tilde{z}] \equiv \tilde{l} + \frac{\mu - a_{11}}{a_{12}} [z(t) - \tilde{z}] \tag{27}$$

$$c(t) - \tilde{c} = \left[\Omega_z l + (\Omega_l + \Omega) \frac{a_{21}}{(\mu - a_{22})} \right] [z(t) - \tilde{z}] \tag{28}$$

Where μ is the negative eigenvalue corresponding to the linearized dynamic system and the expressions Ω_c and Ω_l are the derivatives with respect to c and l. [14].

6. Distribution

Given the homogeneous nature of Eq.(7), preferences can be perfectly aggregated across all agents and as such the aggregate economy's behavior is independent of its distributional properties. The distributive indicators of the neoclassical framework which are described in this section are as originally derived in Garcia-Penalosa and Turnovsky (2006) and modified in Chatterjee and Turnovsky (2012). Details are provided in the appendix.

6.1. Wealth

We assume that at the beginning of a certain time period $t = 0$, all agents are endowed with a measure of capital, $K_{i,0}$. Accordingly, wealth inequality in period $t = 0$ is given by

$$\sigma_{k,0} = \frac{K_{i,0} - K_0}{K_0} = k_{i,0} - 1; \tag{29}$$

where, given the unit mass of agents, K_0 represents the average capital ownership in period $t = 0$. Hence, $k_{i,0}$ indicates relative capital ownership, and $\sigma_{k,0}$ is the coefficient

[14] For greater detail on the behavior of aggregate variables in transition between states, see Turnovsky (2000).

of variation of capital in $t = 0$. In the appendix, we show that its evolution is governed by the following equation;

$$\sigma_k(t) = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(t\right) - \tilde{z}\right]\right]\tilde{\sigma}_k; \tag{30}$$

where δ_1 and δ_2 are as described in Appendix C. The steady-state quantity $\tilde{\sigma}_k$, is similarly related to the time $t = 0$ equivalent by the following;

$$\tilde{\sigma}_k = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(0\right) - \tilde{z}\right]\right]^{-1}\sigma_{k,0}. \tag{31}$$

Dividing Eq.(30) by Eq.(31) then yields

$$\sigma_k(t) = \frac{\left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(t\right) - \tilde{z}\right]\right]}{\left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(0\right) - \tilde{z}\right]\right]}\sigma_{k,0}; \tag{32}$$

which is the indicator we adopt in the numerical simulations to evaluate the evolution of wealth inequality over time. Accordingly, at every point along the transition to a new steady-state, wealth inequality is evaluated relative to its time zero equivalent. Eq.(32) enables us to express the distributive impact of congestion on wealth inequality as follows;

Proposition 2: *Productive infrastructural provisioning induces an increase in long-run wealth inequality. The extent of increase is however a decreasing function of the degree of congestion.*

That infrastructural provisioning leads to an increase in wealth inequality is due to the private capital productivity unlocked by public capital expansion, the proceeds of which are disproportionately appropriated by individuals with greater capital ownership. The role played by congestion is however less immediately apparent. In Appendix C we show that $\delta_1/(\mu - \delta_2) > 0$, which implies that as congestion increases and the distance $(z(0) - \tilde{z})$ shrinks, then the value of $\tilde{\sigma}_k$ converges towards $\sigma_{k,0}$, given that from Eq.(30),

$$\sigma_{k,0} = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(0\right) - \tilde{z}\right]\right]\tilde{\sigma}_k.$$

Accordingly, greater values of congestion will decrease the extent of dispersion associated with rising wealth inequality. This happens due to the substitution of labor for capital associated with higher degrees of congestion. In general, capital-rich individuals supply relatively less labor than their poorer cohorts. Hence, higher degrees of congestion will be associated with lower average labor supply. Moreover, due to diminishing marginal productivity, this results in greater average productivity of labor, the gains from which will be disproportionately appropriated by the capital-poor, leading to a decrease in wealth inequality.

6.2. Labor Income

Proposition 3: *Through a substitution of capital for labor, congestion externalities induce a decline in labor income inequality; and through increasing the opportunity cost of leisure, congestion externalities induce an increase in labor income inequality.*

In Appendix C we derive the distribution of labor income as follows;

$$\sigma_w(t) = \frac{w\left[L_i(t) - L(t)\right]}{wL(t)} \equiv \Gamma\sigma_k(t); \tag{33}$$

where Γ is a positive expression as defined in the appendix which relates the ownership of capital to the supply of labor. Given that $\Gamma > 0$, the foregoing implies that labor is supplied inversely vis-a-vis relative capital ownership. Moreover, a higher average labor supply also implies a greater dispersion in the aspect of income inequality associated with wages.

Appendix C also shows that the impact of congestion on Eq.(33) can be decomposed into a negative wealth effect and a positive factor substitution effect. Intuitively, congestion leads to a substitution of capital for labor in production, hence increasing equilibrium leisue supply. This disproportionately benefits the capital-poor, resulting in decreasing inequality. However, the negative effect of congestion on wealth makes leisure expensive for capital-rich agents such that they are now compelled to supply more labor which invariably increases aggregate labor supply and lowers its productivity, hence increasing wage inequality.

6.3. Pre-Tax & Post-Tax Income

We define the following measures of pre-tax and after-tax relative income;

$$y_i = \frac{Y_i}{Y} = \frac{\omega K\left(1-l_i\right)+rK_i}{\omega K\left(1-l\right)+rK} \tag{34}$$

$$y_i^a = \frac{\left(1-t_w\right)\omega K\left(1-l_i\right)+\left(1-t_k\right)rK_i}{\left(1-t_w\right)\omega K\left(1-l\right)+\left(1-t_k\right)rK} \tag{35}$$

from which we derive the following expression for pre-tax income inequality, σ_y in time t;

$$\sigma_y(t) = \rho(t)\sigma_k(t); \tag{36}$$

where ρ is correspondingly defined as

$$\rho(t) = s_k - (1-s_k)\frac{l(t)}{1-l(t)}\left[1-\frac{G_2}{G_1\tilde{l}}\right]\left[1+\frac{\delta_1}{\mu-\delta_2}\left[z\left(t\right)-\tilde{z}\right]\right]^{-1}. \tag{37}$$

The post-tax equivalents are similarly defined as

$$\sigma_y^a(t) = \rho^a(t)\sigma_k(t); \tag{38}$$

with

$$\rho^a(t) = \left[\rho(t) + \frac{(t_w - t_k)\, s_k}{s_k(1-t_k) + (1-s_k)(1-t_w)}\,(1-\rho(t))\right]; \tag{39}$$

wherein s_k is capital's share of output, which given the Cobb-Douglas function, is constant. Eq.(36) and (38) represent the coefficient of variation for pre-tax and after-tax income respectively. They demonstrate that while pre-tax income inequality is a composite of capital and labor shares, the added component of policy tools work to determine after-tax inequality. In particular, after-tax income inequality is increasing in the share of capital, and decreasing in the capital income tax. Our final proposition accordingly addresses the limits of fiscal policy in redistribution;

Proposition 4: *Given the presence of capital and labor income taxes, a degree of congestion exists beyond which the capital income tax becomes ineffective in decreasing long-run income inequality.*

Eq.(39) indicates that after-tax inequality is greater or less than its pre-tax equivalent depending on the effects of policy redistribution. The equation portrays the role of policy in directly influence inequality through redistribution rather than indirectly through the effect of gross factor returns. There are unique implications for output and distribution from utilizing each of the distortionary taxes in financing growth; moreover, these implications can be significantly altered in an environment of congested externalities of public capital. To illustrate this, note that the channels of impact for gross factor returns vary significantly by instrument type, as indeed the government budget constraint from Eq.(16) can be rewritten as

$$t_w(1-\alpha) + t_k(R(1-\alpha)+\alpha) + t_c\frac{(1-\alpha)}{(1+t_c)\eta}\frac{l}{1-l} + \tau = \theta \tag{16b}$$

which suggests that a lump-sum tax would proportionally fund the existing share of government capital investment in output. A wage tax similarly would imply that regardless of the level of congestion in the economy, a fixed rate $dt_w/d\theta = 1/(1-\alpha)$. Capital income and consumption taxes will however vary depending on the level of congestion, as can be seen from the fact that in the case of a tax on capital gain, the required rate is given by $dt_k/d\theta = 1/(R(1-\alpha)+\alpha)$; which falls as congestion increases. The intuition for this result being that a higher level of congestion is consistent with an increase in the productivity of capital in the economy, which coupled with the resulting abundance of capital, implies that lower tax rates become sufficient to meet the government's budgetary objectives. Similarly, given that $dl/dR > 0$, higher levels of congestion will imply lower consumption tax rates to fund public capital.

The overall implication of the above for income inequality is two-fold; first, post-tax inequality is less than pre-tax inequality only if a capital tax is in place; second, from Eq. (39) where both taxes are active, the requisite tax on capital income must be such that $t_k > t_w$, a condition which is only met where the following holds

$$1 > \frac{R(1-\alpha)+\alpha}{(1-\alpha)};$$

which clearly becomes more difficult to satisfy as the degree of congestion increases.

6.4. Welfare

Using the instantaneous utility function, we define the following measure of relative welfare dispersion x;

$$x_i^{1/\gamma(1+\eta)} = u = 1 + \phi(\tilde{k} - 1) \tag{43}$$

Where $1 > \phi > 0$ is defined in the appendix. This metric enables us to describe variations in utility from the dispersion of wealth as follows;

$$\sigma_u = \phi\tilde{\sigma}_k \tag{44}$$

In Appendix C , we establish three key results relating to welfare inequality. First, expansionary fiscal policy unambiguously increases welfare inequality; second, congestion restricts the dispersion of welfare; and third, the distribution of welfare is instantaneous in response to fiscal policy changes, and is as such not subject to transitional variations.

7. Numerical Simulation

The benchmark parameterization for the numerical simulation are presented on Table (1).

Table 1: Benchmark Parametization

Preference	γ=-1.5, β=0.04, η=1.75
Production	A=0.6, α=0.4
Externalities	ε=0.4, h=0, R_c=0
Fiscal	θ=0.05, τ=0.05

The values utilized above are generally consistent with those used in the extant literature, moreover Edwards (1990)'s empirical assessment of congestion produced a similar range. Setting γ at -1.5 generates an inter-temporal elasticity of substitution of 0.4, in line with empirical literature while the elasticity of leisure in utility at 1.75 generates a value of steady-state leisure within the bracket of 0.714 and 0.755, consistent with the results of Cooley (1995). Setting the total factor productivity coefficient $A = 0.6$ moderates the scale of productivity to yield plausible growth rates while consistent with Guenven (2006)'s empirical results, we set the rate of time preference at 4%. The flexibility accorded by including the elasticity scaling factor, ε is seen in the fact that setting its value at 0.4, consistent with Boom and Ligthart (2010) enables a plausible variation between the elasticities of private capital at 0.4, the externality at 0.36, and government's capital input in production at 0.24. For the values of relative congestion in production, R, and utility Rc, we evaluate at the parameter values of 0, 0.25 and 0.5 in line with similar studies on congestion such as Pintea and Turnovsky

(2006), Eicher and Turnovsky (2000), and Chatterjee and Ghosh (2011). In order to isolate the channel of impact, we begin by setting h and $Rc = 0$ and subsequently assess outcomes generated by consumption externalities.

Our procedure for generating the model's dynamics involves the inducement of a fiscal stimulus which simultaneously balances the government's budget, we consequently observe and analyze the dynamic adjustments generated therefrom. Accordingly, we frame the government's objective as being to increase its share of output from a subsisting 5% which is financed by a lump-sum tax, to 8% which is to be financed by either of the policy instruments at its disposal, i.e. the capital tax, consumption tax, labor income tax and lump-sum taxes.

The initial steady-state equilibrium values of the public-to-private capital ratio, leisure, output to capital ratio, and growth rate are depicted on table (2) for representative congestion rates of 0, 0.25 and 0.5.

Table 2: Benchmark Steady-State Equilibrium

Financing Policy	Congestion	$\tilde{z}$	$\tilde{l}$	$\tilde{y}$	$\tilde{\psi}$
Lump-sum tax	R=0	0.531	0.714	0.243	2.29%
	R=0.25	0.348	0.736	0.209	3.01%
	R=0.5	0.258	0.755	0.186	3.62%

Table (2) indicates that private output in the congestion-free environment is generated with private capital being twice the amount of public capital in production; this is further combined with about a third of agents' available hours deployed to working which results in an aggregate growth rate of 2.29%; consistent with current economic estimates for developed economies. Progressively however, higher levels of congestion monotonically decreases the public to private capital ratio and increases steady-state leisure consistent with the factor-substitution effect; moreover, the last column indicates progressively higher levels of steady-state GDP growth based on equilibrium overaccumulation.

The first three columns on table (3) indicate the tax differential required to fund the desired increase in government's share evaluated at congestion levels of 0, 0.25 and 0.5. At each level, we assume that if the increase is financed by a specific distortionary tax then all other distortionary taxes are set at zero and no change occurs in the lump-sum tax ratio; if however the increase is financed by an increase in the lump-sum tax, then all distortionary taxes are set at zero.

Accordingly, a constant rate funds the desired increase under the lump-sum and labor-tax financing schemes regardless of the extent of congestion. However, a progressively lower rate is required to meet the tax objective under capital and consumption tax regimes. Consequently, the absolute changes in the economy's public-to-private capital ratio and leisure at the increasing levels show that except for the labor tax financing scheme which leads to an increase in steady-state leisure, all other funding mechanisms are consistent with a decline in leisure at a rate which is monotonically decreasing for higher levels of congestion with higher public-to-private capital ratios. Moreover, the lump-sum tax and consumption tax in general generate values within the same margin, while the lump-sum tax, not surprisingly, produced the highest growth rate in the absence of congestion. Overall a monotonic relationship subsists in the steady-state aggregate changes which indicates that the lump-sum tax rate performs the best for growth; moreover, all tax-types result in higher growth following the increase in public capital.

On Table (4) we present the distribution effects. The values are generated in percentage changes relative to their pre-shock realizations. Considering first the set of rows which display the wealth effect of the expansionary fiscal policy, higher values of steady-state public to private capital levels are associated with higher levels of wealth inequality; moreover, the calibration results also indicate that higher levels of congestion are associated with lower levels of wealth inequality.

Next we highlight the effect of the tax base and congestion on the short-run and steady-states of income inequality. With the exception of the labor tax, a consistent outcome is demonstrated in which the short run is associated with a decrease in income inequality following the stimulus. Underlying this pattern is the fact that the stimulus increases the productivity of both private capital and labor, hence raising the short-run real wage and labor supply which given the disproportionately higher supply by capital-poor agents, causes a decline in income dispersion in the short-run.

Table 3: Steady State Equilibrium Effects

Policy Change	Tax Differential			d$\tilde{z}$			d$\tilde{l}$			d$\tilde{\psi}$		
	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5
Lump-sum tax	0.0300	0.0300	0.0300	0.258	0.178	0.135	-0.009	-0.008	-0.007	0.206	0.190	0.181
Capital income tax	0.0750	0.0545	0.0428	0.353	0.219	0.158	-0.064	-0.006	-0.005	0.100	0.118	0.126
Labor income tax	0.0500	0.0500	0.0500	0.268	0.183	0.138	0.017	0.002	0.003	0.168	0.155	0.146
Consumption tax	0.0366	0.0326	0.0294	0.265	0.181	0.137	-0.002	-0.002	-0.002	0.179	0.168	0.161

Table 4: Distributional Effects Reported As Percentage Changes

	Wealth			Income						Welfare		
	$d\tilde{\sigma}_k$			$d\sigma_y(0)$			$d\tilde{\sigma}_y$			$d\tilde{\phi}$		
	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5	R=0	R=0.25	R=0.5
Lump-sum tax	2.077	1.84164	1.647	-1.382	-1.65523	-1.839	4.323	3.7195	3.288	4.74	4.29015	3.916
Capital income tax	2.7	2.24064	1.936	-7.787	-5.69422	-4.663	-0.945	0.43723	1.03	2.79	2.32398	2.01
Labor income tax	2.136	1.88519	1.682	1.174	0.555277	0.098	7.231	6.21419	5.468	2.326	2.04938	1.8
Consumption tax	2.243	1.97849	1.765	-1.834	-2.00936	-2.132	4.235	3.65951	3.244	2.266	2.00416	1.793

The decline in short-run income inequality is reinforced by the fact that labor jumps in response to the increased wage whereas capital-rich agents are unable to respond in kind owing to the gradually accumulating nature of capital, however in transition they start to yield disproportionate benefits from the increased capital productivity in the form of wealth increases which, coupled with the diminishing returns to the productivity of labor, implies that the decline in income inequality is reversed as the system approaches steady state.

In addition to the factor productivity effect however, distortionary tax financing implies a factor impingement associable with the underlying tax scheme. Where the labor income tax is used for instance, the combination of the wage and labor supply effects highlighted above are insufficient to generate a short-run decline in inequality implying an increase in both short-run and long-run income inequality. Indeed, the result that a tax on labor generates income inequality regardless of the timing horizon is persistent across the levels of congestion itemized on table (4), it however is not robust to sufficiently high levels of congestion as the large volume of capital in production sufficiently strengthens the productivity of labor such that the initial response of income inequality to the financing stimulus initially decreases as indicated on Fig.(6).

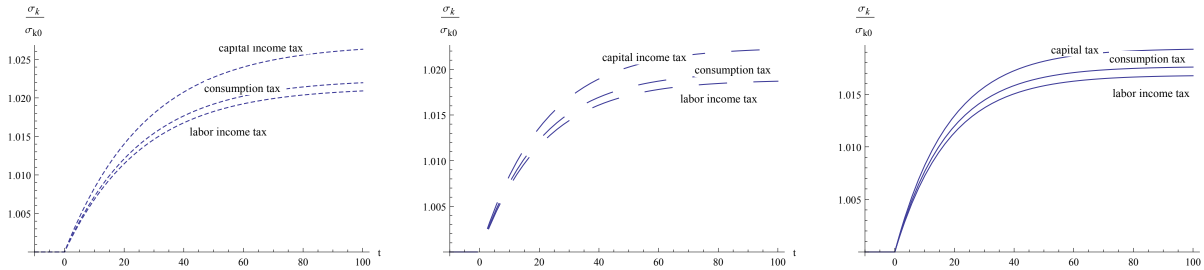
When the financing tool is a capital income tax, the result that inequality declines in both the short and long run is only consistent with the absence of congestion, as indeed the productivity effect of the expansion coupled with the enhanced output elasticity of capital and the wage appreciation jointly induces an eventual increase in income inequality as is indicated when evaluated at the 0.25 and 0.5 congestion levels, again this is evident from Fig's.(2) and (3) where we further see that the recovery pace of capital is increasingly strengthened as congestion intensifies.

The final set of rows highlight the impact of the expansionary policy on welfare inequality given that such is generated on any of the highlighted tax instruments. All welfare variations are compressed with the existence of congestion, regardless of the financing tool. Nonetheless the fiscal expansion is seen to imply an unambiguous increase in the economy's welfare dispersion.

8. Inequality and Growth

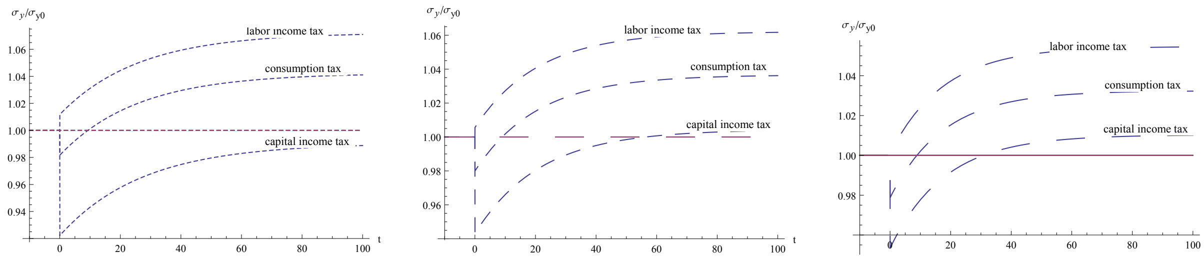
Table 5 summarizes the key effects of the short-run and long-run relationships between inequality and growth calibrated at congestion levels of 0 and 0.5 respectively.

The impact of congestion on growth in the short run is positive regardless of the financing scheme as indeed the percentage changes in growth are moderately higher on comparing columns (2) and (3). Over the same horizon, the lump-sum and consumption taxes exert a stronger reverse effect in the presence of congestion so that the



(a) R=0 (b) R=0.25 (c) R=0.5

Figure 2: Wealth Inequality



(a) R=0 (b) R=0.25 (c) R=0.5

Figure 3: Income Inequality

Table 5: Growth and Income Inequality

Policy Change	Short-Run Change (%)				Long-Run Change (%)			
	$d\psi(0)$		$d\sigma_y(0)$		$d\tilde{\psi}$		$d\tilde{\sigma}_y$	
	R=0	R=0.5	R=0	R=0.5	R=0	R=0.5	R=0	R=0.5
Lump-sum tax	0.136	0.137	-1.382	-1.839	0.206	0.181	4.323	3.288
Capital income tax	0.037	0.086	-7.787	-4.663	0.1	0.126	-0.945	1.03
Labor income tax	0.1	0.103	1.174	0.098	0.168	0.146	7.231	5.468
Consumption tax	0.11	0.118	-1.834	-2.132	0.179	0.161	4.235	3.244

instantaneous decrease in inequality is more pronounced following the deployment of these instruments hence the implication that in the short run a non-distortionary tax monotonically generates lower degrees of inequality and higher growth with higher levels of congestion following the fiscal expansion. The foregoing results with respect to income inequality are sustained in the long run as we see that the eventual level of inequality, though higher in both cases, is less pronounced where congestion exists. This is however not the case for growth as the long-run now evidences lower percentage growth rate changes for the economy evidencing congestion, hence the conclusion that it raises the short-run growth rate and lowers it in the long-run whilst decreasing the severity of income inequality. The qualitative results in both cases however remain unchanged.

When the expansion is financed by a distortionary tax however, the results with a labor income tax channels the above result with the singular exception that in the short-run income inequality actually increases following the fiscal expansion, so that given this financing mechanism, inequality is a constant feature of both short and long runs with an increasing severity over time. As has been assessed earlier however, a sufficiently high level of congestion suffices to reverse this implication in the short-run.

The results with a tax on capital however demonstrates sharp contrasts and over certain ranges highlights the potential for ambiguity in the relationship between inequality and growth given that the instantaneous effect is a decrease in inequality but the long-run effect depends on the extent of congestion in the economy, so that over higher values of congestion, the relationship between inequality might be clearly defined as monotone increasing. What remains clear however both for high and low congestion levels is that the economy never replicates the instantaneous decrease in inequality as it progresses towards steady-state hence reflecting the persistence of wealth and income inequality in the economy. Accordingly it suffices that *the increasing incidence of congestion in an economy leads invariably towards a positive correlation between growth and inequality in the long run, regardless of the financing scheme deployed*. This is in contrast with one of the main findings of Chatterjee and Turnovsky (2012).

Active Consumption Externalities

On Table 6 we present the steady-state macroeconomic and distribution outcomes for the model where both h and R_c are active in the economy and the increase in θ is financed with either a lump-sum tax or a tax on capital income. The results for the first scenario are presented on the first three rows for $R = 0$. The first row shows a moderately higher equilibrium leisure as compared to Table 2 which corresponds to lower income inequality and lower steady-state economic growth given $h = 0.1$. As R_c increases to 0.25 and 0.5 however, growth increases due to a greater value of capital in the economy. The resulting increase in equilibrium labor also means an increase in long-run income inequality. The most distinctive information emerging from the first three columns however is the fact that at higher levels of R_c, initial and steady-state income inequality are conversely related as can be observed by comparing σ_{y0} and σ_y.

When a tax on capital income is used to finance the increase in θ, the key result arising from the second set of rows on Table 6 is that the capacity of t_k to reduce long-run income inequality abrades for higher values of R and R_c and is enhanced for higher values of h. Note also that although both R and R_c result in higher income inequality, the channel through which this happens is completely different. The key to observing this is in the value of equilibrium leisure which is very high for high values of R based on factor substitution, and low for high values of R_c based on the increased supply of labor to access utility-enhancing infrastructure.

Table 6: Calibrated Values With Congestion Externalities in Consumption

R	R_c	h	z_0	l_0	y_0	ψ_0	z	l	y	ψ	σ_K	σ_{y0}	σ_y	ϕ
Lump-sum Tax Financed														
0	0	0.1	0.58	0.716	0.247	2.13	0.864	0.71	0.277	2.57	2.43	-1.38	4.32	1.52
0	0.25	0.1	0.51	0.713	0.241	2.37	0.767	0.7	0.271	2.82	2.64	-2.66	4.54	1.85
0	0.5	0.1	0.454	0.71	0.236	2.6	0.69	0.7	0.266	3.1	2.84	-2.8	4.58	9.26
Capital Income Tax Financed														
0	0.5	0.5	0.33	0.7	0.224	3.356	0.598	0.696	0.259	3.47	8.7	-17	-1.1	9.95
0	0	0.5	0.77	0.72	0.262	1.7	0.77	0.717	0.314	1.588	5.93	-23	-8	9.39
0.5	0.5	0.5	0.2	0.75	0.235	4.36	0.353	0.743	0.269	4.68	5.27	-11.6	1.67	10

Policy Implications

The foregoing dynamics have been driven by the distribution of factor ownership and the impact of productivity shocks thereon. In the use of the labor-income tax, while the dueling implications arising from the factor return versus the redistributive effects repose a prevailing dominance for the latter effect hence a positive relationship between inequality and growth, it is possible to separate both components through a policy mix which simultaneously limits excess supply entry of labor and raises overall economic productivity via a lump-sum tax. By implication the endogenous response of the labor-leisure choice becomes the primary determinant of the relationship between inequality and growth hence adducing the possibility of growth alongside falling income inequality regardless of the extent of congestion in the economy.

Funding infrastructure upgrade through a tax on capital income has the potential to raise long-run inequality depending on the extent of productivity boost occasioned by policy and the impact of the capital elasticity of output on the tax rate, whereas both the taxes on consumption and the lump-sum tax generate a strong pure productivity effect which unambiguously increases inequality in the long run with a weaker but persistent effect attributable to congestion. Accordingly, a policy hybrid which simultaneously strengthens the productivity of labor and constrains the excess labor supply appears to provide the sole mechanism through which growth is generated alongside a decrease in income inequality regardless of the extent of congestion

9. Empirical Evaluation

Our next set of results establish how well empirical variables corroborate the calibrated outcomes. Our approach here is to seek out the closest fit between observable macroeconomic data and the parameters underlying the model with especial emphasis on R, R_c, and h. To do this, we compile averages on $C/Y, l, z, \psi, K/Y$ and C/Y for the US, UK, Japan, Germany and Canada over the period 1960-2009 as presented on Table (7), and compare the resulting data to the model's predictions under two scenarios; one in which infrastructure is financed out of a non-distortionary tax, and the second in which the same extent of infrastructure is financed out of a distortionary tax. Data on pubic and private capital as well as GDP comes from the IMF database while all other data was obtained from the Penn World Tables.

The above listed variables appear largely in unison across the five economies. The

Table 7: Country Averages of Relevant Variables- 1960-2009

Country	C/Y	l	z	ψ	θ	K/Y	C/K
Germany	0.57	0.80	0.33	0.03	0.04	4.27	0.13
Canada	0.58	0.79	0.36	0.03	0.04	3.63	0.16
Japan	0.49	0.77	0.72	0.04	0.09	3.69	0.14
United Kingdom	0.66	0.80	0.41	0.02	0.04	4.44	0.15
United States	0.65	0.79	0.58	0.03	0.05	3.53	0.18

most significant source of variation appears however to be the public to private capital ratio, which varies from a high of 72% of private capital in Japan to 33% in Germany. Whereas domestic funding platforms with respect to infrastructural challenges are expectedly heterogeneous, an aspect of this variability may be due to the nature of assembling the data given that various sources provide conflicting data. The IMF indicates that the average z level between 1960 and 2015 for the US stands at 0.58; prior work on this ratio from Lansing (1995) however suggests it has declined steadily from about 0.45 in 1947 to a present level of around 0.2. This caveat is made to draw attention to the fact that data sources may influence the overall nature of the pattern discernible from the data.

Table 8: Data-Based Calibrations

Variable	BM	Sim 1	Sim 2	Sim 3	Sim 4	Sim 5
R	0	0.5	0	0	0	0.5
h	0	0	0.1	0.1	0	0.1
R_c	0	0	0	0.5	0	0.5
z	44	28	48	37	46	20
C/Y	87	82	87	85	87	81.6
C/K	20	20	20	19	19	18
K/Y	430	410	420	446	436	580
ψ	2.1	3.3	1.9	2.3	2.0	3
θ	4	4	4	4	4	4
l	71	76	72	71	72	76
t_w	0	0	0	0	2.1	2.1
t_c	0	0	0	0	2.1	2.1
t_k	0	0	0	0	2.1	2.1
t	4	4	4	4	0	0

Table 8 presents the results of the data-based calibrations, with outcomes rendered in percentages. As a benchmark, we set all externalities and distortionary taxes equal to zero and compare the results to the data. The results as provided under BM, indicate that in the model economy without congestion externalities or distortionary taxes, the equilibrium public to private capital ratio is 44% which are reasonably comparable with the average across the listed economies of 48%, while output grows at 2% and consumers maximize lifetime utility by consuming 87% of currently produced output.

Whereas the benchmark model may be interpreted as an attainable framework for any of the listed economies in the absence of fiscal distortions and externalities, we can expect there to always be a wedge between the model's benchmark and empirically observed outcomes for at least two reasons. First, is the absence of frictions in the model economy. The literature on precautionary savings which arises due to imperfect capital markets and stochastic growth frontiers suggests that in the presence of such frictions, the consumption to output ratio will be significantly lower as agents adjust to the possibility of future uninsurable shocks. Secondly, we have modeled a closed economy; this implies that economies experiencing persistently high net exports will be overcompensated by the model[15].

The externalities which support production are an apparent source of over-accumulation, as the next set of simulations captioned *Sim 1* with the congestion parameter R set to 0.5 results in a lower C/Y ratio, and higher output growth and equilibrium leisure. The results for the second and third simulations evaluate the role of consumption externalities with h set at 0.1. The results herein are more comparable to the data in terms of the growth rate of output, but less so in terms of equilibrium hours worked since agents now value leisure less. This implies that we can account for a significant degree of the variations between $z, C/K$ and ψ by a conditioning of the degree of congestion between production and consumption externalities.

The next set of results create policy-based distortions by admitting the existence of consumption, capital and labor income taxes. In the first simulation, we assume that

[15]There is a similar argument to be made for the variation between equilibrium leisure in the data at 79% of total worker hours and the model prediction of 71% as arising due to the data's inclusion of involuntary idle time in the form of unemployment.

they equally fund the government's operation with no distortions due to externalities. These are presented in the column labelled *Sim 4*, and they produce a capital to output ratio of 4.36, which is highly comparable to the average across the five economies of 4.30. There however remains the challenge of a low growth rate implied by this combination of parameters, which are addressed in the final calibration exercise by assuming that all policy instruments exist amidst production and consumption externalities.

This final calibration exercise provides the closest fit to the data in terms of equilibrium $l, \psi, C/K$ and C/Y ratios. It also provides an approximation to Lansing (1995)'s z ratio discussed earlier. The difficulty associable with this simulation however lies in its apparent over-estimation of the capital to output ratio K/Y as compared to the data. This implies that some intangible accumulation is required to sustain a 3% growth rate but keep the K/Y ratio contained at the empirically evaluated 4.3[16]. Overall however, our comparison of the data to the model's outcomes suggest an improvement of fit in line with the existence of congestion externalities.

10. A Sensitivity Analysis

In this section, we evaluate the sensitivity of the primary results to variations in the substitutability of the productive inputs. As indeed, it is plausible to argue that infrastructure provision may induce greater income inequality whenever producers are more (or less) inclined to substitute away from labor as the extent of congestion increases. Accordingly, we re-specify Eq.(1) in the form of a CES production function as follows;

$$Y_j = A[\alpha K_j^{-\mu} + (1-\alpha)(L_j X_j)^{-\mu}]^{-\frac{1}{\mu}} \tag{45}$$

[16]The likely candidate for such intangible accumulation is human capital.

Where the elasticity of substitution is given by

$$\zeta = \frac{1}{1+\mu} \tag{46}$$

Optimizing on the productive inputs then produces the following equilibrium factor prices;

$$r = A^{-\mu}(R(1-\alpha)+\alpha)y^{\mu+1} \tag{47}$$

$$w = A^{-\mu}z^{-\mu\epsilon}\alpha\left(\frac{y}{(1-l)}\right)^{\mu+1} \tag{48}$$

Our emphasis is therefore on the elasticity of substitution, ζ, which varies between zero, in the case where both factors are perfectly substitutable, and infinity, wherein they are perfect complements. Our strategy accordingly will be to calibrate the model for a low value of the elasticity of substitution, corresponding to $\zeta = 0.5$, and a high equivalent, corresponding to $\zeta = 1.2$;.

As a benchmark, we evaluate the impact of congestion on inequality when infrastructure is financed using a non-distortionary lump-sum tax. The results are presented on Table 9. On the upper panel, we present outcomes for the low elasticity state, while the lower panel shows the high elasticity equivalent following the expansion in infrastructure. The outcomes are consistent with the unitarily elastic case given that following the increase in θ, income inequality initially falls and then rises over time regardless of the nature of elasticity. Moreover, higher congestion levels consistently induce greater levels of economic growth and increases welfare inequality.

The sole pattern of divergence observable between high and low substitutability of factors arises from its impact on wealth inequality, which is negative when factors are highly substitutable, and positive when $\zeta = 1.2$ Moreover, from the first column, it is apparent that this outcome is not due to congestion but is exacerbated by it.

Table 10 produces calibration outcomes where the upgrade is financed by a capital income tax in both low and high substitutability conditions. The third and fourth

Table 9: Lump-Sum Tax-Financed Infrastructure, $\zeta = 0.5, 1.2$

	$\zeta = 0.5$		
R	0	0.25	0.5
σ_k	-0.08	-0.3	-0.5
σ_u	0.4	1.8	1.6
$\sigma_{y(0)}$	-3.5	-3.2	-2.9
$\tilde{\sigma}_y$	12.7	12.2	11.7
ψ	1.4	1.7	1.9
	$\zeta = 1.2$		
σ_k	3.2	3	2.8
σ_u	6	5.6	5.2
$\sigma_{y(0)}$	-0.8	-1.3	-1.8
$\tilde{\sigma}_y$	2.8	2.7	2.5
ψ	3.2	4.2	5.1

rows confirm the results presented in the case of unitary elasticity, given that in all circumstances except where $R = 0; \zeta = 1.2$, income inequality rises in the long run despite the activation of a capital income tax.

Accordingly, we highlight two distinct outcomes from this exercise. First is a re-iteration of the underlying relationship between congestion, inequality and growth. Secondly, as the elasticity of substitution between capital and labor decreases, the degree of congestion required to abrade the redistributive efficacy of the capital income tax lessens.

Table 10: Capital Income Tax-Financed Infrastructure, $\gamma = 0.5, 1.2$

	$\zeta = 0.5$		
R	0	0.25	0.5
σ_k	-0.14	-0.6	-0.8
σ_u	0	-0.5	-0.7
$\sigma_{y(0)}$	-23	-18	-15
$\tilde{\sigma}_y$	3.6	4.7	5
ψ	1.3	1.6	1.8
	$\zeta = 1.2$		
σ_k	4	3.6	3.3
σ_u	4.1	3.7	3.4
$\sigma_{y(0)}$	-5.5	-4.1	-3.7
$\tilde{\sigma}_y$	-1.2	0.4	1.2
ψ	3	4	4.9

11. Conclusion

In assessing the implications of public capital vis-a-vis the relationship between inequality and growth, we have re-framed the resulting discourse within the context of the severity of congestion so that lower levels of congestion under a financing scheme with an active capital income tax has the tendency to decrease income inequality while enhancing growth whereas at higher levels of congestion, both inequality and growth increase. Under alternative financing schemes however, income inequality is seen to bear a monotonic relationship with growth in the long run. Moreover, the effect of government policy is seen to be characterized by sharp tradeoffs with a faster consolidation speed associable with higher degrees of congestion. Furthermore, in terms of the implication of the foregoing for welfare, our results demonstrate that the dual implications of higher growth and lower wealth concentration increases average welfare and decreases its underlying dispersion.

Accordingly, the ambiguity in the extant literature regarding the relationship between growth and inequality can be addressed. Once this appropriation of productive externalities is incorporated moreover the inequality implications for economies are apparent. A high capital/output ratio contrary to empirical observations may be inevitable as the source of capital accumulation incentive in society are not taken into account. Congestion externalities in the present model deals with this and achieves an empirically validated capital-to-output ratio.

Finally, we acknowledge that the results above are described for an economy with complete markets and the absence of the human capital input, the inclusion of which may significantly enrich the results obtained herein. Further research may accordingly proceed along these lines.

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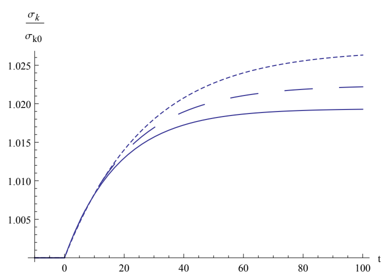
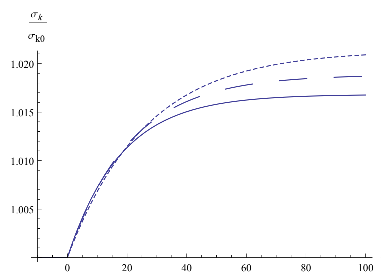
Appendix

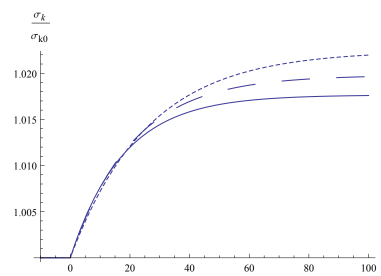
Appendix A. Dynamics



capital income tax

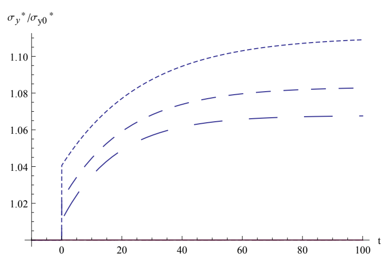


labor income tax



consumption tax

Wealth Inequality



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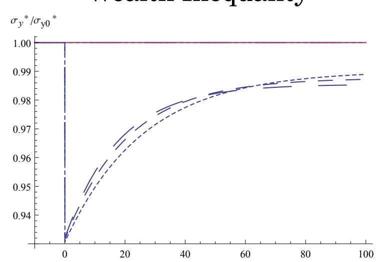
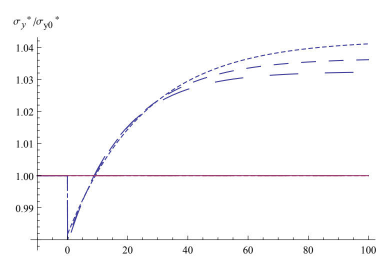
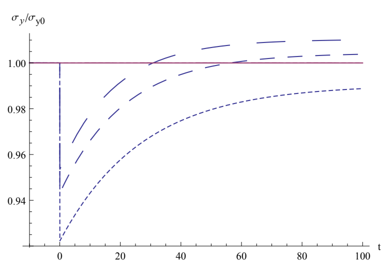


labor income tax

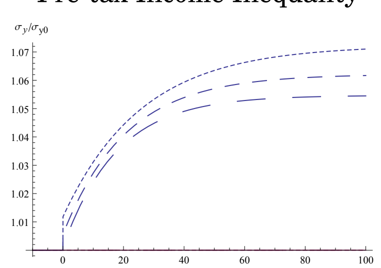


consumption tax

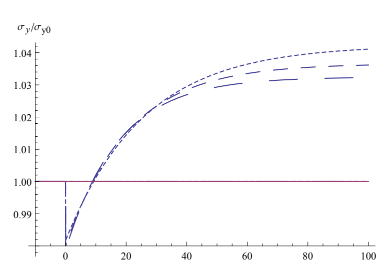
Pre-tax Income Inequality



capital income tax

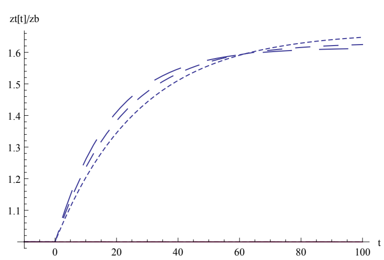


labor income tax

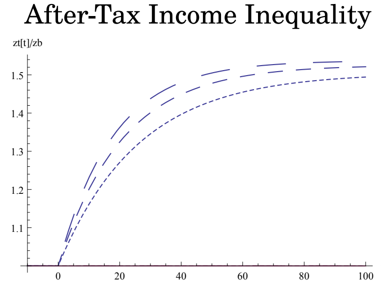


consumption tax

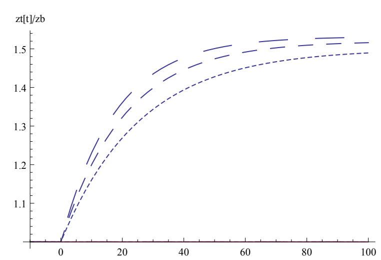
After-Tax Income Inequality



capital income tax

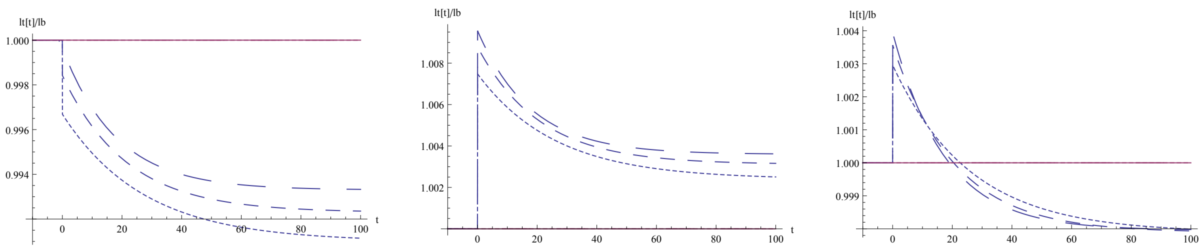


labor income tax



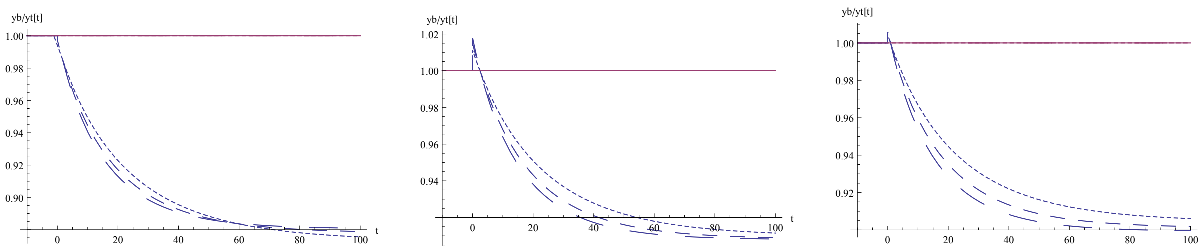
consumption tax

Public/Private Capital



capital income tax | labor income tax | (a) consumption tax

Leisure Dynamics



capital income tax | labor income tax | consumption tax

Capital/Output Ratio Dynamics

Appendix B. The Linearized Matrix

Details of the linearized matrix given in equation (25) are indicated below;

$$a_{11} = \tilde{y}_z\theta + \tilde{y}\left[\frac{(1-t_w)(1-\alpha)}{(1+t_c)\,\eta}\frac{\tilde{l}}{1-\tilde{l}} - (1+R(1-\alpha)-\theta)\right]$$

$$a_{12} = \tilde{y}_l\left[\frac{(1-t_w)(1-\alpha)}{(1+t_c)\,\eta}\frac{\tilde{l}}{1-\tilde{l}} - (1+R(1-\alpha)\right] + \frac{(1-t_w)\,(1-\alpha)\,\tilde{y}\tilde{z}}{(1+t_c)\,\eta\left(1-\tilde{l}\right)^2}$$

$$a_{21} = \begin{Bmatrix} \left[R(1-\alpha)(\gamma-t_k)+(1-t_k)\alpha+(1-\gamma)\left(\theta-1+\dfrac{(1-t_w)(1-\alpha)}{(1+t_c)\,\eta}\dfrac{l}{1-l}\right)\right]y_z \\ -\varepsilon(1-\gamma)(1-\alpha)\dfrac{a_{11}}{z}+G_3 \end{Bmatrix}\frac{1}{H}$$

$$a_{22} = \begin{Bmatrix} (1-t_k)(R(1-\alpha)+\alpha)y_l-(1-\gamma)\left[y_l(1+R(1-\alpha))-\dfrac{(1-t_w)\,(1-\alpha)}{(1+t_c)\,\eta\,(1-l)}\left(y_l l+\dfrac{y}{1-l}\right)\right] \\ -\varepsilon(1-\gamma)(1-\alpha)\dfrac{a_{12}}{z}+G_4 \end{Bmatrix}\frac{1}{H}$$

$$G_3 = hR_c\,(1-\alpha)\,(1-t_w)\,\tfrac{y_z l}{1-l}+h\eta\gamma\theta\left(\tfrac{zy_z-y}{z^2}\right)$$

$$G_4 = hR_c\,(1-\alpha)\,(1-t_w)\,\tfrac{y_l l+y}{(1-l)^2}+h\eta\gamma\theta\tfrac{y_l}{z}$$

Appendix C. Distribution

Appendix C.1. Wealth

Combining Eq.s(8) and (20), we can express the evolution of relative capital k_i, defined as K_i/K in the following way

$$\dot{k}_i = \left[\frac{\dot{K}_i}{K_i} - \frac{\dot{K}}{K}\right] k_i = \omega(1-t_w)\left(1 - l_i - \frac{l_i}{\eta}\right) - \tau y - \left(\omega(1-t_w)\left(1 - l - \frac{l}{\eta}\right) - \tau y\right) k_i \quad (A1)$$

Which, given restrictions imposed by the transversality condition, is an unstable differential equation. Define;

$$G_1 \equiv (1+t_c)\Omega + (1-t_w)\omega; \; G_2 \equiv (1-t_w)\omega - \tau y$$

So that Eq.(A1) can be re-expressed as

$$\dot{k}_i(t) = (G_2 - G_1 l_i(t)) + (G_1 l(t) - G_2)\, k_i(t)$$

Or more conveniently as a linear deviation from the average wealth.

$$\dot{k}_i(t) = -G_1\,(l_i(t) - l(t)) + (G_1 l(t) - G_2)\,(k_i(t) - 1) \quad (A2)$$

Where $G_1 \geq G_2$. We will use the expression G_{1x} to indicate the derivative of G_1 with respect to x. Setting $\dot{k} = 0$ produces the following steady-state relationships;

$$\tilde{l}_i - \tilde{l} = \left[l - \frac{G_2}{G_1}\right](\tilde{k}_i - 1) \quad (A3a)$$

$$\pi_i - 1 = \left[1 - \frac{G_2}{G_1 \tilde{l}}\right](\tilde{k}_i - 1) \quad (A3b)$$

Linearizing Eq.(A2) around steady-state yields;

$$\dot{k}_i(t) = \delta_1 \left(\tilde{k}_i - 1\right) [z(t) - \tilde{z}] + \delta_2 \left[k_i(t) - \tilde{k}_i\right] \tag{A4}$$

Where;

$$\delta_1 \equiv \frac{\partial \dot{k}_i(t)}{\partial l}\frac{dl}{dz} + \frac{\partial \dot{k}_i(t)}{\partial z} = \frac{G_{1z}G_2}{G_1} - G_{2z} + \left(\frac{G_2}{l} + \frac{G_{1l}G_2}{G_1} - G_{2l}\right)\left(\frac{a_{21}}{\mu - a_{22}}\right)$$

$$\delta_2 \equiv \frac{\partial \dot{k}_i(t)}{\partial k_i} = G_1 l - G_2$$

where, from the transversality condition, $\delta_2 > 0$.

In order to determine the sign of δ_1, we note that G_1 and G_2 are both homogeneous in y. This means we can re-write the expressions as; $G_1 = Ay$ and $G_2 = By$ where A and B are given by;

$$A = \frac{(1-t_w)(1-\alpha)}{\eta} + \frac{(1-t_w)(1-\alpha)}{(1-l)}$$

$$B = \frac{(1-t_w)(1-\alpha)}{(1-l)} - \tau$$

Then, the term

$$\frac{G_{1z}G_2}{G_1} - G_{2z}$$

can be expressed as

$$\frac{(G_{1,z}G_2 - G_1G_{2z})}{G_1} = \frac{(ABy - AyB)\,\partial y/\partial z}{G_1} = 0$$

And similarly,

$$\frac{G_{1l}G_2}{G_1} - G_{2l} = \frac{(G_{1,l}G_2 - G_1G_{2l})}{G_1} = \frac{(ABy - AyB)\,\partial y/\partial l}{G_1} = 0$$

Hence, the expression for δ_1 reduces to

$$\delta_1 = \frac{G_2}{l}\left(\frac{a_{21}}{\mu - a_{22}}\right)$$

Fulfilling the transversality condition requires that $G_2/l > 0$, while saddle-path stability requires that $a_{21}/(\mu - a_{22}) < 0$. Accordingly, $\delta_1 < 0$.

Using the stable eigenvalue, the solution to Eq.(A4) is then given by

$$k_i(t) - \tilde{k}_i = \frac{\delta_1}{\mu - \delta_2}\left(\tilde{k}_i - 1\right)[z(t) - \tilde{z}] \qquad (A5)$$

Note that since $\delta_1 < 0, \delta_2 > 0$ and the stable eigenvalue $\mu < 0$, it therefore follows that $\delta_1/(\mu - \delta_2) > 0$.

Given Eq.(A5), we now express relative capital in terms of its coefficient of variation

$$\sigma_k(t) = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(t\right) - \tilde{z}\right]\right]\tilde{\sigma}_k \qquad (A6a)$$

$$\tilde{\sigma}_k = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(0\right) - \tilde{z}\right]\right]^{-1}\sigma_{k,0} \qquad (A6b)$$

Dividing Eq.(A6a) by Eq.(A6b) yields Eq(32) in the text, i.e.

$$\sigma_k(t) = \frac{\left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(t\right) - \tilde{z}\right]\right]}{\left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z\left(0\right) - \tilde{z}\right]\right]}\sigma_{k,0}$$

Appendix C.2. Labor

Using Eq.(A3), relative wage can be expressed as;

$$w_i(t) - 1 = \frac{w\left[L_i(t) - L(t)\right]}{wL(t)} = \frac{l(t)}{1 - l(t)}\left[1 - \frac{G_2}{G_1\tilde{l}}\right](\tilde{k}_i - 1) \qquad (A7a)$$

this implies

$$\sigma_w = \frac{l(t)}{1-l(t)}\left[1-\frac{G_2}{G_1\tilde{l}}\right]\sigma_k \tag{A7b}$$

Taking the derivative of the right hand side and noting that $dl/dR > 0$. we obtain the following;

$$\frac{d\sigma_w}{dR} = \frac{\sigma_k}{(1-l)}\underbrace{\left\{\frac{dl/dR}{1-l}\left(1-\frac{G_2}{G_1}\right) - l\underbrace{\left(\frac{\overbrace{dG_2/dR}^{-} - \left(\overbrace{G_1 dl/dR}^{+} - \overbrace{l dG_1/dR}^{-}\right)}{(G_1 l)^2}\right)}_{-}\right\}}_{+} + \underbrace{\frac{l\,\overbrace{d\sigma_k/dR}^{-}}{1-l}\left(1-\frac{G_2}{G_1 l}\right)}_{-} \tag{A8a}$$

The effect of congestion depends on whether the negative wealth effect (the second term) dominates the positive production substitution effect (the first term). Congestion increases leisure supply given the increased productivity of capital, which leads to an aggregate decrease in labor supply and an increase in the wage rate. Marginal product of labor increases, the gains of which are appropriated proportionally more by capital poor; however, the negative effect of congestion on wealth makes leisure expensive for capital rich agents such that they are compelled to supply more labor which invariably increases aggregate labor supply and lowers its productivity hence increasing wage inequality.

We also evaluate variations in the dispersion of wealth for changes in government's share of output;

$$\frac{d\sigma_w}{d\theta}=\frac{\sigma_k}{(1-l)}\underbrace{\left\{\frac{dl/d\theta}{1-l}\left(1-\frac{G_2}{G_1 l}\right)-l\underbrace{\left(\frac{\overbrace{dG_2/d\theta}^{+}-\left(\overbrace{G_1 dl/d\theta}^{-}-\overbrace{l dG_1/d\theta}^{+}\right)}{(G_1 l)^2}\right)}_{+}\right\}}_{-}+\underbrace{\frac{l\overbrace{d\sigma_k/d\theta}^{+}}{1-l}\left(1-\frac{G_2}{G_1 l}\right)}_{+} \quad (A8b)$$

The sign directions above indicate that although the increased government share has a productivity augmenting impact for labor, this is appropriated by all agents and since aggregate labor supply increases due to the stimulus, the dispersion in labor income is widened by the expansionary policy thus leading to the negative sign on the first component. However, the expansion implies that capital rich agents now supply even less labor given that they now face a more dispersed relative capital edge which consequently compresses the wage dispersion, hence the positive sign on the second component. When compared with the result from Eq.(A8a), congestion acts in a converse direction to policy, so that its impact in a growing economy would be to decrease the extent of variation associated with wealth inequality during a fiscal expansion and increase it during a contraction given the pure factor supply effect.

Appendix C.3. Welfare

Substituting for C_i from Eq.(13) into the instantaneous utility for agent i in Eq.(7a) and combining with Eq.(A3a) produces the following

$$x_i=\frac{U_i}{U}=\left[1+\left[1-\frac{G_2}{G_1\tilde{l}}\right](\tilde{k}_i-1)\right]^{(1+\eta)\gamma} \quad (A9)$$

Its monotonic transformation yields a metric x for evaluating relative welfare, defined as follows

$$x_i^{1/\gamma(1+\eta)} = u = 1 + \phi(\tilde{k} - 1) \tag{A10}$$

where ϕ is defined as follows

$$\phi = \left[1 - \frac{G_2}{G_1 \tilde{l}}\right] \tag{A11}$$

with the coefficient of variation of relative welfare is given by

$$\sigma_u = \phi \tilde{\sigma}_k \tag{A12}$$

The impact of an expansionary fiscal policy in the presence of congestion can similarly be derived as being

$$\frac{d\tilde{\sigma}_u}{d\theta} = \tilde{\sigma}_k \underbrace{\left[\frac{\overbrace{dG_2/d\theta}^{+} - [G_1 \overbrace{dl/d\theta}^{-} - l \overbrace{dG_1/d\theta}^{+}]}{(G_1 l)^2}\right]}_{+} + \underbrace{\overbrace{d\tilde{\sigma}_k/d\theta}^{+} \left[1 - \frac{G_2}{G_1 \tilde{l}}\right]}_{+} \tag{A13}$$

$$\frac{d\tilde{\sigma}_u}{dR} = \tilde{\sigma}_k \underbrace{\left[\frac{\overbrace{dG_2/dR}^{-} - [G_1 \overbrace{dl/dR}^{+} - l \overbrace{dG_1/dR}^{-}]}{(G_1 l)^2}\right]}_{-} + \underbrace{\overbrace{d\tilde{\sigma}_k/dR}^{-} \left[1 - \frac{G_2}{G_1 \tilde{l}}\right]}_{-} \tag{A14}$$

The above implies that the fiscal stimulus unambiguously increases welfare dispersion while congestion decreases it so that actual dispersion of welfare is typically contained between both margins. Along the transition path welfare inequality remains unchanged so that steady-state dispersion levels are instantly visible upon policy change.

Appendix D. Comparative Statics

To assess the impact of a change in infrastructure on the steady state values of z and l, we rewrite as follows

$$A1 = \frac{\dot{C}}{C}; A2 = \frac{\dot{K}}{K}; A3 = \frac{\dot{K}_G}{K_G}$$

Where $\frac{\dot{C}}{C}$, $\frac{\dot{K}}{K}$, and $\frac{\dot{K}_G}{K_G}$ are as defined in Eq.'s(18), (20), and (15) respectively. Then, totally differentiating the system (23a) and (23b), we obtain;

$$\begin{bmatrix} 1 & -f_{A1,l} & -f_{A1,z} \\ 1 & -f_{A2,l} & -f_{A2,z} \\ 1 & -f_{A3,l} & -f_{A3,z} \end{bmatrix} \begin{bmatrix} d\psi \\ dl \\ dz \end{bmatrix} = \begin{bmatrix} f_{A1,\theta} \\ f_{A2,\theta} \\ f_{A3,\theta} \end{bmatrix} d\theta + \begin{bmatrix} f_{A1,R} \\ f_{A2,R} \\ f_{A3,R} \end{bmatrix} dR + \begin{bmatrix} f_{A1,R_c} \\ f_{A2,R_c} \\ f_{A3,R_c} \end{bmatrix} dR_c$$

Setting the respective initial distortionary taxes equal to zero, from the Jacobian, we obtain the determinant which is expressed as;

$$D = \frac{A^2(1-\alpha)}{\eta z^{2-2(1-\alpha)}(1-\gamma)(1-l)^{2\alpha}} \left(\begin{array}{l} z\varepsilon\left(R(1-\alpha)(1+hR_c\eta)+\alpha+hR_c\eta\right) \\ +\theta \left(\begin{array}{l} +(1-\gamma)(1-(l+\varepsilon)(1-\alpha))+1+hR_c-\gamma \\ -l(1+hR_c(1-\alpha)+\alpha+\gamma+R\gamma-R\alpha\gamma) \\ -(1-\alpha)(R\gamma+h\varepsilon(R_c+R_cz-z\gamma))\eta \end{array} \right) \\ -\eta\theta^2(1-l)(1-\gamma(1-h\eta)) \end{array} \right) \quad (B1)$$

Which is difficult to sign. However, invoking the restrictions implied by Eq.(14) and the non-negativity of Eq.(20), the limits on the resulting parameter restrictions imply $D > 0$.

Accordingly, differentiating the system with respect to R and inspecting the relative parameter magnitudes, we obtain the following;

$$\frac{d\psi}{dR}=\frac{A^{3}\left(1-l\right)\left(1-\alpha\right)^{2}\theta}{D\eta\left(1-\gamma\right)\left(1-l\right)^{3\alpha}z^{2-3(1-\alpha)}}\left(\begin{array}{l}1+hR_{c}\eta+l\eta\theta\left(1+h\gamma\eta\right)+\eta\left(1-\alpha-\theta-h\gamma\eta\theta\right)\\-\varepsilon\left(1-\alpha\right)\left(1+hR_{c}\eta\right)-l\left(1-\alpha\right)\left(1+\eta+hR_{c}\eta\right)\end{array}\right)>0 \tag{B2}$$

$$\frac{dl}{dR}=\frac{A^{2}\left(1-l\right)\left(1-\alpha\right)^{2}}{D\eta\left(1-\gamma\right)\left(1-l\right)^{2\alpha-1}z^{2-2\varepsilon(1-\alpha)}}\left(\begin{array}{l}z\varepsilon(1-\alpha)^{2}\left(l-\eta\left(1-l-hlR_{c}\right)\right)\\-\left(1-l\right)\eta\theta\left(\gamma-\varepsilon\left(1-\alpha\right)\left(z-\gamma\left(1+hz\eta\right)\right)\right)\end{array}\right)>0 \tag{B3}$$

$$\frac{dz}{dR}=\frac{A^{2}\left(1-l\right)\left(1-\alpha\right)^{2}}{D\eta\left(1-\gamma\right)\left(1-l\right)^{2\alpha-1}z^{2-2\varepsilon(1-\alpha)}}\left(\begin{array}{l}z\left(1+\eta+hR_{c}\eta-\alpha\eta-l\left(1-\alpha\right)\left(1+\eta+hR_{c}\eta\right)\right)\\-\left(1-l\right)\eta\left(z+\gamma+hz\gamma\eta\right)\theta\end{array}\right)<0 \tag{B4}$$

Similarly, the derivatives with regard to congested consumption externalities yield;

$$\frac{d\psi}{dR_{c}}=-\frac{A^{3}hl(1-\alpha)^{2}\theta}{D\eta\left(1-\gamma\right)\left(1-l\right)^{3\alpha}z^{2-3(1-\alpha)}}\left(\begin{array}{l}1-\varepsilon\left(1-\alpha\right)+\eta+R\eta-\eta\left(R\alpha+\theta\right)\\-l\left(1-\alpha-R\alpha\eta+\eta\left(1+R-\theta\right)\right)\end{array}\right)>0 \tag{B5}$$

$$\frac{dl}{dR_{c}}=\frac{A^{2}hl\left(1-\alpha\right)}{D\eta\left(1-\gamma\right)\left(1-l\right)^{2\alpha}z^{2-2\varepsilon(1-\alpha)}}\left(\begin{array}{l}z\varepsilon\eta\left(1-\alpha\right)\left(\left(1-l\right)\left(1-R\left(1-\alpha\right)\right)-l\left(1-\alpha\right)\right)\\+\left(1-l\right)\eta\theta\left(1+\left(1-z\right)\left(1-\alpha\right)\varepsilon\right)\end{array}\right)<0 \tag{B6}$$

$$\frac{dz}{dR_{c}}=\frac{A^{2}hl(1-\alpha)^{2}}{D\eta\left(1-\gamma\right)\left(1-l\right)^{1+2\alpha}z^{1-2\varepsilon(1-\alpha)}}\left(\begin{array}{l}z\left(1-l\left(1-\alpha\right)-\eta\left(1-l\right)\left(1+R\left(1-\alpha\right)\right)\right)\\-\left(1-l\right)\left(1+z\right)\eta\theta\end{array}\right)<0 \tag{B7}$$

Where in each case, the anchor behind R_C's potential impact on the system comes from the non-zero realization of h.

Appendix E. Excludable Public Capital

Extending the model to account for excludability per Ott and Turnovsky (2006), implies modifying Eq.(1b) such that the externality is now made up of two components; one which is excludable, and the other which is not. To restrict our focus to the publicly provided externality, we set $\varepsilon = 1$ and denote the productivity associated with the excludable part of the publicly provided externality as σ. The composite externality X_j then assumes the form;

$$X_j = K_E^{\sigma} \left(\frac{K_j}{K}\right)^{R1} K_N^{1-\sigma} \left(\frac{K_j}{K}\right)^{R2} \tag{C1}$$

Where K_E and K_N represent the aspect of public capital that are excludable and non-excludable respectively. $R1$ and $R2$ are also now the extent of congestion associable with the excludable and non-excludable externality.

Output as perceived by the jth producer is now given by

$$Y_j = AK_E^{\sigma(1-\alpha)} K_N^{(1-\sigma)(1-\alpha)} \left(\frac{K_j}{K}\right)^{R1(1-\alpha)} \left(\frac{K_j}{K}\right)^{R2(1-\alpha)} L_j^{(1-\alpha)} K_j^{\alpha} \tag{C2}$$

With the equilibrium marginal product of capital being analogous to Eq.(5) as

$$r = ((R_1 + R_2)(1-\alpha) + \alpha)y \tag{C3}$$

The consumer now maximizes a Hamiltonian which includes two types of publicly availed resources, where the excludable aspect of this externality imposes a user fee, p. Hence, the consumer has to optimize on the quantity of this excludable resource required to maximize utility. This leads to an extra first-order condition;

$$\Omega^a_{KE} \equiv \frac{\partial Y_j}{\partial K_E} = \frac{\sigma(1-\alpha)Y_j}{K_E^D} = p \tag{C4}$$

Here K_E^D now represents the quantity of the excludable capital that the consumer demands. In equilibrium, this quantity demanded will equal the quantity supplied. Taking the quantity as a proportion of output, $K_E = \theta_1 Y$, it then becomes clear that the user fee is a fixed rate

$$p = \frac{\sigma(1-\alpha)}{\theta_1} \tag{C5}$$

Note also that the equilibrium user fee is independent of the extent of congestion in the economy. By this token, the sole difference between the user fee and a flat tax on output is in the voluntary opt-in associated with the user fee. it is however also the case that the productive nature of public capital makes its take-up essentially guaranteed from the perspective of the profit-maximizing producer. It therefore naturally follows that the aggregate and distributional consequences of such an externality in the decentralized economy are also fully described using the budget constraint from Eq.(8) and (16) where p may now be a construed as a component of the proportional tax on output, i.e. τ.

Appendix F. Leisure and Inequality

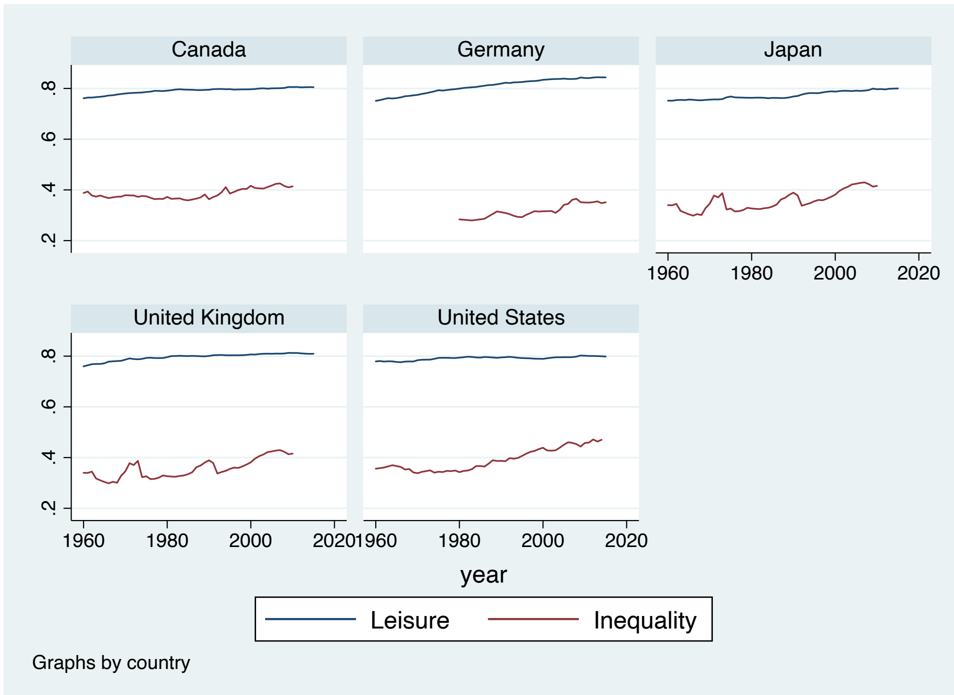


Figure F.6: Leisure Hours and Income Inequality. Income inequality is evaluated using the fraction of output accruing to the top decile. Source: PWT and WIID.