Optimal Risk Sharing with Ex Post Private Information: Rules vs Discretion

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Abstract

We consider the optimal contract under cost uncertainty between a risk averse buyer and a risk averse supplier when the supplier can privately discover more information after contracting but before production. We show that the optimal production schedule is often characterized by distortions and rules which restrict the supplier from adjusting the production schedule to the cost realization. The supplier is constrained by stricter rules in high cost states, but enjoys more discretion to adjust the production schedule in low cost states. Moreover, the supplier is granted with more discretion when the buyer becomes more risk averse, but is restricted by more rules when the supplier's risk preference becomes private information.

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JEL Classification: D81; D82; D86

1 Introduction

Firms often contract in environments of uncertainty. For example, retailers usually are uncertain about market conditions for the upcoming season when contracting with their suppliers; manufacturers often face uncertainty over the future cost of inputs when contracting with the buyers of their products; and health insurers often contract with physicians with uncertain cost of treating patients. In these environments of uncertainty, risk sharing is often an important element of economic contracts.

Theoretically, optimal risk sharing has drawn substantial attention in the agency literature. For example, Zeckhauser (1970), Spence and Zeckhauser (1971), Holmstrom (1979), Shavell (1979), and Grossman and Hart (1983) among others consider optimal risk sharing in the presence of moral hazard problems; Salanie (1990) studies optimal risk sharing in the presence of adverse selection problems; Laffont and Rochet (1998), Theilen (2003), and Dai (2008) study optimal risk sharing under both adverse selection and moral hazard problems. In all these studies, the optimal contracts critically depend on the risk preference of the contracting parties. In reality, however, contracting parties seldom have perfect information on each other's risk preference, and they may have incentive to misrepresent the information. For example, US companies often do not have precise information on the risk preference of their international suppliers, and the suppliers may exaggerate their vulnerability to risk in order to secure more favorable contact terms.

Moreover, in environments of uncertainty, contracting parties often can discover more information after signing a contract but before conducting production. For instance, in the aforementioned examples, after signing their contracts, retailers typically can obtain more knowledge about the market conditions; and manufacturers and healthcare providers often can collect additional information about their costs. Contracts that optimally utilize the arrival of new information not only could be superior in theory, but also have had important applications in practice.[1] In environments of uncertainty, how much discretion should be granted to the risk averse supplier to utilize the new information? How does the utilization of new information depend on the risk preferences of the contracting parties?

The purpose of our study is to investigate how optimal risk sharing and the utilization of new information over time depend on the information asymmetry on risk preference and the relative degree of risk aversion between the contracting parties.

We consider a principal-agent relationship where a risk averse buyer contracts with a risk averse supplier for the production of certain good. At the time of contracting, both the buyer and the supplier are uncertain about the cost of production. However, after signing the contract and before the production, the supplier can privately discover the realization of cost condition.

When the supplier's risk preference is common information, both parties have symmetric information at the time of contracting. Although the supplier can capture information rent after signing the contract due to its ex post private information on the cost realization, the buyer can fully extract the expected information rent at the time of contracting. It is well known that the efficient outcomes could be achieved in this case through a fixed-price contract if the supplier were risk neutral. However, when the supplier is risk averse, a fixed-price contract generally is no longer optimal as it imposes the entire risk on the supplier. The optimal production schedule must balance risk sharing, production efficiency, and the supplier's incentive to truthfully reveal its cost realization. We show that the optimal production schedule is distorted to be less than the efficient level except for the lowest and the highest cost realizations.

Moreover, when the supplier becomes sufficiently risk averse, the optimal contract is characterized by "rules" for high cost realizations. More specifically, the supplier can adjust its output level only for low cost realizations, and is required to produce a constant level

[1] For example, Courty and Li (2000) show how airlines offer different ticketing arrangements to screen flyers who learn their valuation of air travel over time.

of output for a range of high cost realizations. Rules arise as an optimal solution to the conflict among risk sharing, production efficiency, and information revelation. However, when the buyer becomes more risk averse, the supplier is granted with more "discretion" to adjust the production schedule to cost realizations

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its cost realization but also by its degree of risk aversion. Under the optimal contract, the production schedule for the more risk averse supplier is distorted further downwards to limits a less risk averse supplier's incentive to mimic a more risk averse supplier. The supplier's private information on its risk preference also aggregates the conflict among risk sharing, production efficiency, and information revelation. As a result, a more risk averse supplier is granted with even less discretion over its production schedule and rules arise more frequently in its production schedule.

Inflexible rules are commonly observed in vertical relationships. For example, in the apparel industry, retailers are often required to make firm, SKU-specific orders well in advance of the beginning of the selling season despite demonstrable advantages to in-season replenishment; in the electronics industry, flexibility for reorders is often restricted within some prespecified limits of original forecasts (Barnes-Schuster et al., 2002). Procurement contracts with limited flexibility have also been used by Sun Microsystems, Toyota Motor Corporation, IBM, Hewlett Packard, and Compaq. A similar structure, called a "Take-or-Pay" provision, is often embedded in long-term supply contracts for natural resources. (Tsay and Lovejoy 1999). Our analysis suggests that the seemingly inefficient inflexible rules can be an optimal solution to the conflict among risk sharing, production efficiency, and information revelation in these environments of uncertainty.

Lewis and Sappington (1989a, 1989b) among others study the optimality of inflexible rules in agency contracts when agents face "countervailing incentives", i.e., agents have incentive to either understate or overstate their private information depending on the state of nature. In contrast, we show that rules can arise, in the absence of countervailing incentives, as an optimal solution to the conflict among risk sharing, production efficiency, and information revelation. Salanie (1990) and Laffont and Rochet (1998) study optimal procurement contracts between a risk neutral buyer and a risk averse supplier with common information on contract parties' risk preferences. They show that "bunching" arises in the supply contract for the worst states of nature. In contrast, we focus on the optimal risk sharing between a risk averse buyer and a risk averse supplier. We show that, the nature of "bunching" depends on the information asymmetry on risk preference and the relative degree of risk aversion between contracting parties.

de Mezza and Webb (2000) and Jullien, Salanie and Salanie (2007) study the optimal insurance contracts under moral hazard when insurance customers are privately informed of their risk preference. Landsberger and Meilijson (1994) consider the optimal insurance contract between one risk neutral monopolistic insurer and one risk averse agent who is privately informed of his degree of risk aversion. Smart (2000) studies a screening game in a competitive insurance market in which insurance customers differ with respect to both accident probability and degree of risk aversion. In contrast to the above studies, we consider the optimal procurement contract when suppliers differ with respect to both cost of production and degree of risk aversion.

Our study also relates to the literature on dynamic mechanism design. Baron and Besanko (1984), Riordan and Sappington (1987), Courty and Li (2000), Dai et al. (2006), Pavan, Segal, and Toikka (2014)), and Krahmer and Strausz (2015a and 2015b) among others study two-period models where risk neutral agents learn payoff-relevant private information in both periods. They analyze the optimal mechanism where the contract is signed in the first period before the agent discovers his second period private information. In contrast to these articles, we study the optimal contract between risk averse parties. We investigate the interaction among risk sharing, production efficiency, and information revelation in the optimal contract.

The rest of the paper is organized as follows. Section 2 describes the central elements of the model. As a benchmark, Section 3 presents the optimal contract under perfect information. Section 4 examines the optimal contract when the supplier's only private information is the realization of cost condition. Section 5 studies the optimal contract when the supplier is privately informed of its risk preference in addition to its cost realization. Finally, Section 6 summarizes our main findings and concludes the paper with future research directions. The proofs of all formal conclusions are in the Appendix.

2 The model

A buyer contracts with a supplier to obtain some quantity, $q \geq 0$, of a good. The buyer's valuation of q is $V(q)$, and $V(\cdot)$ is a smooth, increasing, and concave function. The buyer's surplus is $W = V(q) - T$, where T is the buyer's payment to the supplier. The supplier's total cost of producing q is $C = cq$, where c is the supplier's marginal/average cost of production. Hence, the supplier's profit is $\pi = T - cq$. The supplier's reservation utility is normalized to be zero.

The utility function for both the buyer and the supplier belong to some smooth one-dimensional family of utility functions $\mathcal{F} = U_\rho(\cdot)$. The function $U_\rho(\cdot)$ is ranked according to the Arrow-Prat measure of risk aversion: $-U_\rho''(x)/U_\rho'(x)$ is increasing with ρ for any wealth level x. Thus, ρ measures the supplier's degree of risk aversion. The supplier's degree of risk aversion is unknown to the buyer. However, it is common knowledge that the supplier's degree of risk aversion, ρ, belongs to the two point support $\{l, h\}$ with $h > l$, $\Pr(\rho = l) = \alpha_l$, and $\Pr(\rho = h) = \alpha_h = 1 - \alpha_l$. The buyer's degree of risk aversion is ρ_b. We denote the buyer's utility function as $U_b(\cdot)$ to simplify the notation.

The supplier's marginal cost of production, c, is uncertain at the time of contracting. However, it is common knowledge that the distribution of c follows an absolutely continuous and strictly increasing cumulative distribution function $F(c)$ on $[\underline{c}, \overline{c}]$. After contracting with the buyer and before the production takes place, the supplier privately discovers the realization of c.

We assume that the distribution of c satisfies the following regularity condition: $d[c + F(c)/f(c)]/dc \geqslant 0$. The condition is commonly imposed in the agency literature to ensure that the equilibrium production schedule to be monotonically decreasing in c. As we demonstrate later in our analysis, the condition does not ensure such property for the optimal production schedule in our setting.

The timing and the contractual relation between the buyer and the supplier are as follows: (1) the supplier privately learns its degree of risk aversion ρ; (2) the buyer offers the supplier a set of contract menus $M_\rho = \{T_\rho(c), q_\rho(c)\}$ conditional on the supplier's degree of risk aversion ρ and its realization of marginal cost c; (3) the supplier selects its preferred menu M_ρ given its private information on ρ; (4) the supplier discovers c, and selects a desired option $(T_\rho(c), q_\rho(c))$ from the selected menu M_ρ; (5) exchange takes place according to the contract terms.

3 Perfect Information

As a benchmark, we first discuss the optimal risk sharing when the buyer has perfect information on the supplier's risk preference as well as the realized cost condition.

Define $W_\rho(c) = V(q_\rho(c)) - T_\rho(c)$ for $\rho = l,\ h$. The buyer's optimization problem is choosing a contract menu $M_\rho = \{T_\rho(c), q_\rho(c)\}$ for $\rho = l,\ h$ to maximize

$$E[U_b] = \int_{\underline{c}}^{\overline{c}} U_b(W_\rho(c)) f(c) dc, \tag{1}$$

subject to the individual rationality constraint. The individual rationality constraint requires the supplier's expected utility from the contract be nonnegative in order to guarantee its participation, that is,

$$E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))f(c)dc \geqslant 0. \tag{2}$$

Proposition 1 describes the general properties of the optimal contract with perfect information.

Proposition 1 *Under perfect information, the optimal contract $\{T_\rho(c), q_\rho(c)\}$ for $\rho = l$, h has the following properties:*

(a) $E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(\pi_\rho(c))dF(c) = 0$;

(b)

$$V'(q_\rho(c)) = c; \tag{3}$$

(c)

$$\frac{U_b'(W_\rho(c))}{B_\rho} = \frac{U_\rho'(\pi_\rho(c))}{S_\rho}, \tag{4}$$

where $B_\rho \equiv \int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(c))dF(c)$ *and* $S_\rho \equiv \int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(c))dF(c)$.

First, without private information on its risk preference, the supplier always receives exactly its reservation utility under the optimal contract. Second, regardless of its risk preference, the supplier is required to produce the first-best (efficient) level of output – the production is at the level that the buyer's marginal valuation of output equals the supplier's marginal cost of output. In other words, the production maximizes the joint surplus for the buyer and the supplier for each realization of c.

Part (c) in Proposition 1 describes the optimal risk sharing under perfect information. Notice that $U_b'(W_\rho(c))$ is the increase in the buyer's utility as a result of one unit increase in her surplus for the realization c, and $B_\rho \equiv \int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(c))dF(c)$ is the increase in the buyer's expected utility as a result of one unit increase in her surplus for all realizations of c. Therefore, the term on the left-hand side of equation (4) is the buyer's certainty equivalent (CE) of one unit increase in her surplus for the realization c. Similarly, $S_\rho \equiv \int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(c))dF(c)$ is the increase in the supplier's expected utility as a result of one unit increase in its profit for all realizations of c, and the term on the right-hand side of equation (4) is the supplier's CE of one unit increase in its profit for the realization c. Optimal risk sharing requires the CE of one more dollar be the same for both the buyer and the supplier for each realization of c.

4 Common Information on Risk Preference

In this section we discuss the optimal contract when the supplier's degree of risk aversion is common information. However, after contracting and before the production takes place, the supplier privately discovers the realization of c.

Notice that in this case both parties have the same information on the cost of production at the time of contracting. When the supplier is risk neutral, the optimal contract is a fixed-price contract which makes the supplier the residual claimant of the production. Specifically, the buyer charges the supplier a fixed payment which equals the buyer's expected surplus from the optimal contract under perfect information, and grants the supplier full discretion over production. As the residual claimant of the production, the supplier will choose to produce the first-best level of output based on the realization of cost condition; and the supplier will receive exactly its reservation utility in expectation. However, since the supplier bears the full consequence of cost uncertainty in a fixed-price contract, it is no longer optimal when the supplier is risk averse. An optimal contract must balance risk sharing, production efficiency, and the supplier's later incentive to reveal its private information on cost realization.

The buyer's optimization problem is choosing $\{T_\rho(c), q_\rho(c)\}$ to maximize its expected surplus:

$$E[U_b] = \int_{\underline{c}}^{\overline{c}} U_b(V(q_\rho(c)) - T_\rho(c))dF(c), \tag{5}$$

for $\rho = l,\ h$.

A contract is feasible (or implementable) provided it is *incentive compatible* and *individually rational*. Incentive compatibility requires that the contract induces each type of supplier to truthfully report its realization of cost condition, i.e.,

$$\pi_\rho(c_i \mid c_i) \geqslant \pi_\rho(c_i \mid c_j) \text{ for } c_i \neq c_j, \tag{6}$$

where $\pi_\rho(c_i \mid c_i)$ and $\pi_\rho(c_i \mid c_j)$ denote the supplier's respective profits from choosing options $(T_\rho(c_i), q_\rho(c_i))$ and $(T_\rho(c_j), q_\rho(c_j))$ when the realization of its marginal cost is c_i. Individual rationality requires that the expected utility from the contract for each type of supplier must be nonnegative, i.e.,

$$E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))dF(c) \geqslant 0. \tag{7}$$

Proposition 2 describes the general properties of the optimal contract when the supplier's risk preference is common information.

Proposition 2 *The optimal contract has the following properties: for $\rho = l,\ h$,*

(a) $E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(\pi_\rho(c))dF(c) = 0$*;*

(b) $\int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(x))V'(q)dF(x)/B_\rho = \int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))cdF(x)/S_\rho$*;*

(c) In no bunching region, $q_\rho(c)$ is given by

$$\frac{U_b'(W_\rho(c))}{B_\rho}[V'(q_\rho(c)) - c]f(c) = \frac{\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)}{B_\rho} - \frac{\int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dF(x)}{S_\rho}. \tag{8}$$

Proof. See appendix. ■

When the supplier's degree of risk aversion is common information, both parties have symmetric information at the time of contracting. Consequently, although the supplier can capture ex post information rent from its private information on the realization of c after signing the contract, the buyer can fully extract the expected information rent at the time of contracting by reducing the level of transfer payments $T(c)$ for all realizations of c. (Note that it is the difference in $T(c)$ that provides the incentive for the supplier to truthfully reveal its marginal cost.) Consequently, under the optimal contract, the supplier receives zero expected utility, and in expectation the buyer's marginal utility from the good equals the supplier's marginal disutility of producing the good .

Given that the buyer can fully extract the supplier's ex post information rent at the time of contracting, the buyer does not face the traditional trade-off between rent extraction and production efficiency as in Baron and Myerson (1982). Instead, the optimal production schedule simultaneously balances risk sharing, production efficiency, and the supplier's incentive to reveal the cost realization. Equation (8) demonstrates the intuition.

When the supplier's realization of marginal cost is $\widetilde{c}$, raising $q_\rho(\widetilde{c})$ by δq will in expectation increase the production efficiency by $\delta q[V'(q_\rho(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})$, and the CE of which for the buyer is $\delta q[V'(q_\rho(\widetilde{c})) - \widetilde{c}]U_b'(W_\rho(\widetilde{c}))f(\widetilde{c})/B_\rho$.

However, the increase in $q_\rho(\widetilde{c})$ will also raise the supplier's ex post information rent by δq when $c < \widetilde{c}$, because the higher $q_\rho(\widetilde{c})$ increases the supplier's information rent from exaggerating its cost condition and choosing the option $(T_\rho(\widetilde{c}), q_\rho(\widetilde{c}))$. Consequently, in expectation the increase in $q_\rho(\widetilde{c})$ raises the supplier's CE of ex post information rent by $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)/S_\rho$. Notice that $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)$ is the increase in the supplier's expected utility as a result of the increase in ex post information rent, and S_ρ is the increase in the supplier's expected utility as a result of one unit of increase in $\pi_\rho(c)$ for all realizations of c. Therefore, $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)/S_\rho$ is the CE of the increase in ex post information rent for the supplier. In anticipation of the supplier's CE of ex post information rent, at the time of contracting the buyer reduces $T_\rho(c)$ for all realizations of c by $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)/S_\rho$. Doing so limits the supplier's expected utility to zero. However, the CE of the increase in ex post information rent for the buyer is $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_b(W_\rho(x))dF(x)/B_\rho$. In summary, the expression on the right hand side of equation (8) captures the buyer's marginal cost of increasing output — the decrease in the buyer's CE when the CEs of the ex post information rent are different for the two parties. At the optimum, the buyer's marginal benefit of raising $q_\rho(\widetilde{c})$ must equal its marginal cost of doing so, which yields equation (8).

When both parties are risk neutral, both $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_b(W_\rho(x))dF(x)/B_\rho$ and $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)/S_\rho$ become $\delta q F(\widetilde{c})$, which means the CE of the increase in ex post information rent is $\delta q F(\widetilde{c})$ for both the buyer and the seller. Consequently, the buyer can extract the supplier's expected ex post information rent by reducing the transfer payments for all realizations of c by exactly $\delta q F(\widetilde{c})$. In that case, the right-hand side of equation (8) becomes zero, and $V'(q_\rho(c)) = c$. The optimal contract would be a fixed price contract under which the supplier is granted with full discretion and always produces the efficient level of output.

On the other hand, production inefficiency arises when both parties are risk averse. As we demonstrate below, the risk preference of each party has profoundly different effects on the interaction among risk sharing, production efficiency, and information revelation.

To best demonstrate the effect of each party's risk preference, from this point forward we assume that the supplier's marginal cost, c, follows a uniform distribution on on $[\underline{c}, \overline{c}]$

and define $\Delta c \equiv \bar{c} - \underline{c}$. We first discuss a benchmark case where the buyer is risk neutral but the supplier is risk averse, then consider the general case where both parties are risk averse.

4.1 A risk neutral buyer

When the buyer is risk neutral, $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dx/B_\rho \Delta c$, the buyer's CE of the increase in ex post information rent as a result of raising $q_\rho(c)$ by one unit, is $F(c)$. When the supplier is of small degree of risk aversion, the optimal production schedule is strictly decreasing in c in $[\underline{c}, \bar{c}]$. Then, equation (8) suggests that $\int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho \Delta c = F(c)$ and $V'(q_\rho(c)) = c$ at $\underline{c}$ and $\bar{c}$. In other words, the supplier produces the efficient amount of output at $\underline{c}$ and $\bar{c}$. Notice that $\partial U_\rho'(\pi_\rho(c))/\partial c = -U_\rho''(\pi_\rho(c))q_\rho(c) > 0$. Therefore, the sign of $1 - U_\rho'(\pi_\rho(x)/S_\rho$ must change once and only once with $1 - U_\rho'(\pi_\rho(\underline{c})/S_\rho > 0$ and $1 - U_\rho'(\pi_\rho(\bar{c})/S_\rho < 0$. Since $F(c) - \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho \Delta c = 0$ at $\underline{c}$ and $\bar{c}$, it suggests that $F(c) - \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho \Delta c > 0$ on $(\underline{c}, \bar{c})$. Consequently, equation (8) suggests that the supplier produces less than the efficient level of output on $(\underline{c}, \bar{c})$.

When the supplier becomes sufficiently risk averse, the monotonicity condition ($q_\rho(c)$ is non-increasing) becomes constraining. In this case, rules arise as an optimal solution to the conflict among risk sharing, production efficiency, and information revelation. In other word, bunching occurs and the supplier is required to produce a constant level of good in some interval $[c', \bar{c}]$ where $\underline{c} < c' < \bar{c}$. [2]

Corollary 1 *When the buyer is risk neutral,*

(a) the production schedule is at the first-best level at $\underline{c}$ and $\bar{c}$ but below the first-best level for $c \in (\underline{c}, \bar{c})$ if the supplier is not very risk averse;

[2] With a constant absolute risk aversion (CARA) utility function and an uniform distribution of c, Salanie (1990) and Laffont and Rochet (1998) also show that complete bunching arises in some interval $[c^*, \bar{c}]$ where $\underline{c} < c^* < \bar{c}$.

(b) there exists some σ such that bunching occurs in the production schedule for some interval $[c', \bar{c}]$ if $\rho > \sigma$ for the supplier.

When the supplier converges to infinitely risk averse, $\int_{\underline{c}}^{c} U'_\rho(\pi_\rho(x))dx/S_\rho \Delta c$ converges to zero for $c \in [\underline{c}, \bar{c})$. Then equation (8) converges to

$$[V'(q_\rho(c)) - c]f(c) = F(c) \tag{9}$$

for $c \in [\underline{c}, \bar{c})$, which is the well known solution for a standard adverse selection problem where the supplier is privately informed of its marginal cost of production at the time of contracting. This is because the supplier will participate in the contract only if it is guaranteed nonnegative profit for all realizations of c when it is infinitely risk averse, and the buyer cannot extract any of the supplier's ex post information rent at the time of contracting (as the CE of the ex post information rent is zero for an infinitely risk averse supplier). Consequently our setting becomes equivalent to one that the supplier is perfectly informed of its cost realization at the time of contracting.

4.2 A risk averse buyer.

When the buyer is risk averse, $U'_b(W_\rho(c))$ is increasing in c in an optimal contract, as $W_\rho(c)$ decreases in c and $\partial U'_b(W_\rho(c))/\partial c = U''_\rho(W_\rho(c))W'_\rho(c) > 0$. In words, the buyer's marginal utility from one more dollar increases in an optimal contract as the cost realization increases. Moreover, since $\int_{\underline{c}}^{c} U'_b(W_\rho(x))dF(x)/B_\rho$ equals 0 at $\underline{c}$ and equals 1 at $\bar{c}$, there must exist some cutoff point $\widehat{c} \in (\underline{c}, \bar{c})$ such that $U'_b(W_\rho(c))/B_\rho > 1$ if $c > \widehat{c}$ and $U'_b(W_\rho(c))/B_\rho < 1$ when $c < \widehat{c}$. This introduces two effects on the tradeoff among risk sharing, production efficiency, and information revelation as shown in equation (8).

First, the marginal effect of production efficiency on the buyer's expected utility decreases as c decreases. Compared with a risk neutral buyer, a risk averse buyer places more weight on the marginal production efficiency in high cost (low profit) states but less weight on the marginal production efficiency in low cost (high profit) states.

Second, $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho$, the buyer's CE of the increase in ex post information rent as a result of raising $q_\rho(c)$ by one unit, equals $F(c)$ at $\underline{c}$ and $\overline{c}$, but is less than $F(c)$ at $c \in (\underline{c}, \overline{c})$. In words, as the buyer becomes risk averse, the CE of the ex post information rent as a result of raising $q_\rho(c)$ becomes smaller, because a risk averse buyer's marginal utility of one more dollar decreases as c decreases. Consequently, a risk averse buyer's marginal cost of increasing output – the difference in CEs of the ex post information rent between the two parties – becomes smaller compared with that of a risk neutral buyer.

As a result of the above effects, compared with a risk neutral buyer, the downward production distortion decreases when the buyer becomes risk averse. Consequently, the monotonicity condition ($q_\rho(c)$ is non-increasing) becomes less constraining, and bunching is less likely to arise in equilibrium.

As the buyer becomes more risk averse, everything else equal, $U_b'(W_\rho(x))$ increases more rapidly in c. Then, the difference in CEs of the ex post information rent between the two parties further decreases. When the buyer becomes infinitely risk averse, the optimal contract converges to a fixed price contract: the buyer charges a fixed payment and receives a surplus constant in c; and the supplier is granted with full discretion and produces the efficient level of output.

Corollary 2 *When the buyer is risk averse,*

(a) the production schedule is at the first-best level at $\underline{c}$ and $\overline{c}$ but below the first-best level for $c \in (\underline{c}, \overline{c})$ when the supplier is relatively not very risk averse;

(b) there is less production distortion compared with a risk neutral buyer.

(c) the supplier is granted with more discretion compared with the case of a risk neutral buyer.

For later use, we call the optimal production schedule when the supplier's degree of risk aversion is common information the second-best production schedule.

5 Asymmetric Information on Risk Preference

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its cost condition but also by its degree of risk aversion.

The optimal contract must balance the buyer's expected surplus from different types of suppliers, in addition to the tradeoff among risk sharing, production efficiency, and the incentives for the supplier to truthfully reveal both its cost realization and its degree of risk aversion.

The buyer's optimization problem is choosing a set of contract menus $M_\rho = \{T_\rho(c), q_\rho(c)\}$ for $\rho = l$, h to maximize

$$E[U_b] = \frac{1}{\Delta c} \int_{\underline{c}}^{\overline{c}} \{\alpha_l U_b(W_l(c)) + \alpha_h U_b(W_h(c))\}\, dc \tag{10}$$

subject to

$$E[U_\rho(M_\rho)] = \frac{1}{\Delta c} \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c)) dc \geqslant 0; \tag{11}$$

$$\pi_\rho(c_i \mid c_i) \geqslant \pi_\rho(c_i \mid c_j) \text{ for } c_i \neq c_j; \text{ and} \tag{12}$$

$$E[U_\rho(M_\rho)] \geqslant E[U_\rho(M_s)], \tag{13}$$

where $\rho = l$, h, $s = l$, h, and $\rho \neq s$.

The conditions (11) and (12) ensure the supplier's participation and its truthful revelation of cost realization regardless of its degree of risk aversion; and condition (13) guarantees the supplier truthfully reveals its degree of risk aversion.

Proposition 3 describes the properties of the optimal contract when the supplier is privately informed about its risk preference in addition to the cost realizations.

Proposition 3 *When the supplier is privately informed about its risk preference in addition to the cost realizations, the optimal contract has the following properties:*

(a) $E[U_l(M_l)] > E[U_h(M_h)] = 0$;

(b) In no bunching region, the optimal supply schedule for the less risk averse supplier is characterized by

$$\frac{\alpha_l U_b'(W_l(c))[V'(q_l(c)) - c]}{\alpha_l B_l + \alpha_h B_h} = \frac{\alpha_l B_l}{\alpha_l B_l + \alpha_h B_h} \left[\frac{\int_{\underline{c}}^{c} U_b'(W_l(x))dx}{B_l} - \frac{\int_{\underline{c}}^{c} U_l'(\pi_l(x))dx}{S_l} \right]; \tag{14}$$

and the optimal supply schedule for the more risk averse supplier is characterized by

$$\begin{aligned} \frac{\alpha_h U_b'(W_h(c))[V'(q_h(c)) - c]}{\alpha_l B_l + \alpha_h B_h} &= \frac{\alpha_h B_h}{\alpha_l B_l + \alpha_h B_h} \left[\frac{\int_{\underline{c}}^{c} U_b'(W_h(x))dx}{B_h} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dx}{S_h} \right] \qquad (15) \\ &\quad + \frac{\alpha_l B_l}{\alpha_l B_l + \alpha_h B_h} G(c), \end{aligned}$$

where

$$G(c) \equiv \frac{\int_{\underline{c}}^{c} U_l'(\pi_h(x))dx}{S_l} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dx \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{S_h S_l}.$$

Proof. See Appendix. ■

Under the optimal contract, the buyer can fully extract the more risk averse supplier's ex post information rent by adjusting the level of payments for all realizations of cost condition as in the case of common information on risk preference. However, the utility function of a less risk averse supplier is an increasing and convex transformation of that of a more risk averse supplier, and the less risk averse supplier can always enjoy positive expected utility by mimicking a more risk averse supplier. Consequently, the optimal contract provides a less risk averse supplier positive expected utility to induce its truthful revelation of its degree of risk aversion.

Under the optimal contract, the production schedule for the less risk averse supplier optimally balances risk sharing, production efficiency, and the supplier's incentive to reveal its cost realization, as in the case of common information on risk preference. For the less risk averse supplier, when the realization of marginal cost is $\widetilde{c}$, raising $q_l(\widetilde{c})$ by δq will increase $W_l(\widetilde{c})$ by $[V'(q_l(\widetilde{c})) - \widetilde{c}]\delta q/\Delta c$ which increases the buyer's certainty equivalent by $\delta q \alpha_l U_b'(W_l(\widetilde{c}))[V'(q_l(\widetilde{c})) - \widetilde{c}]/(\alpha_l B_l + \alpha_h B_h)\Delta c$. Note that $\alpha_l B_l + \alpha_h B_h$ is the increase in the buyer's expected utility resulting from one unit increase in its surplus for all possible events. On the other hand, the increase in $q_l(\widetilde{c})$ will also raise the less risk averse supplier's ex post information rent by δq when $c < \widetilde{c}$. For the buyer, the additional ex post information rent is equivalent to a reduction of $\delta q \int_{\underline{c}}^{\widetilde{c}} U_b'(W_l(x))dx/B_l\Delta c$ in $W_l(c)$ for all realizations of c. For the less risk averse supplier, however, the CE of the additional ex post information rent is $\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_l(x))dx/S_l\Delta c$. As discussed earlier, at the time of contracting the buyer can optimally reduce the payment by $\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_l(x))dx/S_l\Delta c$ for all realizations of c. Consequently, as shown in equation (14), the buyer faces the same tradeoff as in the case of common information on risk preference, and the less risk averse supplier is required to produces according to the second-best production schedule.

For the more risk averse supplier, equation (15) demonstrates how the optimal production schedule balances risk sharing, production efficiency, incentives for information revelation, and the buyer's surplus from different types of suppliers.

Increasing $q_h(\widetilde{c})$ by δq increases the buyer's CE by $\delta q \alpha_h U_b'(W_h(\widetilde{c}))[V'(q_h(\widetilde{c})) - \widetilde{c}]/(\alpha_l B_l + \alpha_h B_h)\Delta c$. However, the increase in $q_h(\widetilde{c})$ also increases the ex post information rent for both types of suppliers. For the buyer, the additional ex post information rent is equivalent to a reduction of $\delta q \int_{\underline{c}}^{\widetilde{c}} U_b'(W_h(x))dx/B_h\Delta c$ in $W_h(c)$ for all realizations of c. For the more risk averse supplier, the CE of the additional ex post information rent is

$\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_h(x))dx/S_h\Delta c$. Therefore, the additional ex post information rent eventually reduces the buyer's surplus from a more risk averse supplier by $\delta q[\int_{\underline{c}}^{\widetilde{c}} U_b'(W_h(x))dx/B_h - \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_h(x))dx/S_h]/\Delta c$ for all realizations of c. In addition, the increase in $q_h(\widetilde{c})$ will also raise the less risk averse supplier's rent by $\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_h(x))dx/\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dx$ when it mimics the more risk averse supplier. Then the buyer must also increase the less risk averse supplier's payments by $G(\widetilde{c})\delta q$ for all realizations of c to induce its truthful revelation of its risk preference. Consequently, the increase in $q_h(\widetilde{c})$ also reduces the buyer's surplus from a less risk averse supplier by $G(\widetilde{c})\delta q$ for all realizations of c. Therefore, the right-hand side of (15) is the overall effect of the additional ex post information rent on the buyer's certainty equivalent. Notice that the ratio $\alpha_\rho B_\rho/(\alpha_l B_l + \alpha_h B_h)$ for $\rho = l, h$ measures how the risk averse buyer weights the surpluses from different types of suppliers.

The term $\alpha_l B_l G(c)/(\alpha_l B_l + \alpha_h B_h)$ (which is positive on $(\underline{c}, \overline{c})$ as shown in the proof of Corollary 3) captures the extra distortion due to the supplier's private information on its risk preference. In order to limit a less risk averse supplier's incentive to exaggerate its degree of risk aversion, the buyer further distorts the more risk averse supplier's production schedule. Consequently, as we show in Corollary 3, the more risk averse supplier produces below the second-best production schedule.

Corollary 3 *Under the optimal contract, the more risk averse supplier's production schedule is below the second-best level.*

Proof. See Appendix. ■

For a simple example, suppose that one type of supplier is risk neutral and the other type of supplier is infinitely risk averse. When the buyer is risk neutral, equation (15) becomes

$$\alpha_h[V'(q_h(c)) - c]f(c) = F(c). \tag{16}$$

A direct comparison between equations (9) and (16) demonstrates the effect on the optimal contract of the supplier's private information on its risk preference. An increase in $q_h(\widetilde{c})$ by δq increases the more risk averse supplier's production efficiency by $[V'(q_h(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})\delta q$ regardless whether the supplier is privately informed of its risk preference. However, with private information on risk preference, an increase in $q_h(\widetilde{c})$ by δq increases the ex post information rent for not only the more risk averse supplier but also the less risk averse supplier by $F(\widetilde{c})$. The CE of the ex post information rent is zero for the more risk averse supplier, which means that the buyer cannot extract any of the ex post rent at the time of contracting. Consequently, an increase in $q_h(\widetilde{c})$ by δq increases the expected information rent by $F(\widetilde{c})\delta q$ overall. The comparison between equations (9) and (16) shows that the buyer further distorts the more risk averse supplier's contract towards a cost plus contract compared with the case of common information on risk preference.

When the buyer becomes risk averse, equation (16) becomes

$$\frac{\alpha_h U_b'(W_h(c))[V'(q_h(c)) - c]}{(\alpha_l B_l + \alpha_h B_h)\Delta c} = \frac{\alpha_h B_h}{\alpha_l B_l + \alpha_h B_h} \frac{\int_{\underline{c}}^{c} U_b'(W_h(x))dx}{B_h \Delta c} + \frac{\alpha_l B_l}{\alpha_l B_l + \alpha_h B_h} F(c), \quad (17)$$

and the second term on the right-hand side of (17) captures the effect of asymmetric information regarding risk preference. The extra distortion due to the asymmetric information becomes smaller compared to the case of a risk neutral buyer as $B_l < B_h$, i.e., the distortion for a more risk averse supplier becomes more costly and the information rent for a less risk averse supplier becomes less important to a risk averse buyer.

Corollary 4 *The more risk averse supplier is granted with less discretion compared with the case of common information on risk preference.*

Proof. See Appendix. ■

As either type of supplier becomes sufficiently risk averse, the monotonicity condition

($q_\rho(c)$ is non-increasing) becomes constraining and rules arise in the optimal contract, similar to the case of common information on risk preference. Then, the supplier is required to produce a constant level of output in some interval $[c'_\rho, \bar{c}]$ where $\underline{c} < c'_\rho < \bar{c}$ and $\rho = l,\ h$.

However, the supplier's private information on its risk preference aggregates the conflict among risk sharing, production efficiency, and information revelation. More specifically, for the more risk averse supplier, if bunching occurs in the production schedule in the case of common information on risk preference, it must also occur in the case of asymmetric information on risk preference; however, if bunching occurs in the supply schedule in the latter case, it may not occur in the former case. Therefore, the more risk averse supplier is granted even less discretion over its output level and rules arise more frequently in the case of asymmetric information on risk preference.

Figure 1 illustrates the optimal production schedules for different types of suppliers where bunching occurs for the more risk averse supplier for $c \in (c'_h, \bar{c}]$.

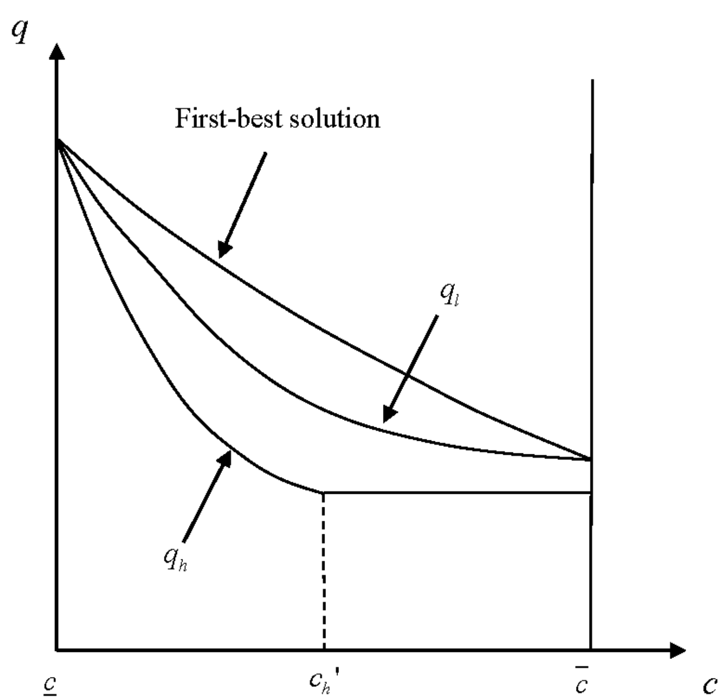

Figure 1. The optimal production schedules for different types of suppliers

6 Conclusion

Firms often contract in environments of uncertainty, and risk sharing is an important element of many economic contracts. We study the optimal risk sharing when the supplier is privately informed of its degree of risk aversion and can privately discover new cost information after contracting but before production.

We show that the optimal contract simultaneously balances risk sharing, production efficiency, information revelation, and the buyer's expected surplus with different types of suppliers. The optimal production schedule is often characterized by distortions and rules which restrict the supplier from adjusting the production schedule to the cost realization. The supplier is constrained by stricter rules in high cost states, but enjoys more discretion to adjust the production schedule in low cost states. Moreover, the supplier is granted with more discretion when the buyer becomes more risk averse, but is restricted by more rules when the supplier's risk preference becomes private information. Our analysis suggests that seemingly inefficient rules commonly observed in vertical relationships can be an optimal mechanism in environments of uncertainty.

Our research could be extended in several directions. For example, although the supplier's information on cost condition is incomplete at the time of contracting, the supplier could be better informed of its future costs than the buyer is. Moreover, suppliers with different levels of expertise might have different forecasts of future costs at the time of contracting. The optimal contracts in these situations merit further investigation.

7 Appendix

7.1 Proof of Proposition 1

The buyer's optimization problem can be written as an optimal control problem with state variables $T_\rho(c)$ and $q_\rho(c)$ and control variables $T'_\rho(c)$ and $q'_\rho(c)$.

The Hamiltonian is

$$H = U_b(V(q_\rho) - T_\rho)f(c) + \mu T'_\rho(c) + \lambda q'_\rho(c) + \theta U_\rho(T_\rho(c) - cq_\rho(c))f(c). \tag{A1}$$

The necessary conditions are given by

$$\frac{\partial H}{\partial T'_\rho(c)} = \mu = 0; \tag{A2}$$

$$\frac{\partial H}{\partial q'_\rho(c)} = \lambda = 0; \tag{A3}$$

$$\lambda' = -\frac{\partial H}{\partial q_\rho} = -[U'_b(W_\rho(c))V'(q_\rho) - \theta U'_\rho(\pi_\rho)c]f(c); \tag{A4}$$

$$\mu' = -\frac{\partial H}{\partial T_\rho} = -[-U'_b(W_\rho(c)) + \theta U'_\rho(\pi_\rho)]f(c); \text{ and} \tag{A5}$$

$$\lambda(\underline{c}) = \lambda(\overline{c}) = \mu(\underline{c}) = \mu(\overline{c}) = 0. \tag{A6}$$

From (A5) and (A6), we have $\mu(\overline{c}) - \mu(\underline{c}) = \int_{\underline{c}}^{\overline{c}}[U'_b(W_\rho(c)) - \theta U'_\rho(\pi_\rho)]dF(c) = 0$. Therefore,

$$\theta = \frac{\int_{\underline{c}}^{\overline{c}} U'_b(W_\rho(c))dF(c)}{\int_{\underline{c}}^{\overline{c}} U'_\rho(\pi_\rho(c))dF(c)} > 0. \tag{A7}$$

From conditions (A3) and (A6), we have $\lambda' = 0$ on $[\underline{c}, \overline{c}]$. Then condition (A4) provides

$$\frac{U_b'(W_\rho(c))V'(q_\rho)}{\int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(c))dF(c)} = \frac{U_\rho'(\pi_\rho(c))c}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(c))dF(c)}. \tag{A8}$$

From conditions (A2) and (A6), we have $\mu' = 0$ on $[\underline{c}, \overline{c}]$. Then condition (A5) provides

$$\frac{U_b'(W_\rho(c))}{\int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(c))dF(c)} = \frac{U_\rho'(\pi_\rho(c))}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(c))dF(c)}. \tag{A9}$$

Equations (A8) and (A9) together provide $V'(q_\rho(c)) = c$.

Moreover, condition (A7) implies $\int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))dF(c) = 0$.

7.2 Proof of Proposition 2

A well known characterization of feasible contracts is the following: *(a)* $T_\rho'(c) = cq_\rho'(c)$; *(b)* $q_\rho(c)$ *is non increasing;* (c) $EU_\rho \geq 0$.

Therefore, the buyer's optimization problem can be written as an optimal control problem with state variables $T_\rho(c)$ and $q_\rho(c)$ and control variables $T_\rho'(c)$ and $q_\rho'(c) = z_\rho$:

$$Max \int_{\underline{c}}^{\overline{c}} U_b(V(q_\rho(c)) - T_\rho(c))dF(c), \text{ subject to} \tag{A10}$$

$$q_\rho'(c) = z_\rho; \tag{A11}$$

$$T_\rho'(c) = cz_\rho; \tag{A12}$$

$$q_\rho'(c) \leqslant 0; \text{ and} \tag{A13}$$

$$\int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))dF(c) \geqslant 0. \tag{A14}$$

The Hamiltonian is

$$H = U_b(V(q_\rho) - T_\rho)f(c) + \mu c z_\rho + \lambda z_\rho + \theta U_\rho(\pi_\rho)f(c). \tag{A15}$$

The necessary conditions are given by

$$\begin{aligned}
\frac{\partial H}{\partial z_\rho} &= \mu c + \lambda \geqslant 0,\ z_\rho \leqslant 0, \text{ and } (\mu c + \lambda)z_\rho = 0; && \text{(A16)}\\
\lambda' &= -\frac{\partial H}{\partial q_\rho} = -[U_b'(W_\rho(c))V'(q_\rho) - \theta U_\rho'(\pi_\rho)c]f(c); && \text{(A17)}\\
\mu' &= -\frac{\partial H}{\partial T_\rho} = -[-U_b'(W_\rho(c)) + \theta U_\rho'(\pi_\rho)]f(c); \text{ and} && \text{(A18)}\\
\lambda(\underline{c}) &= \lambda(\overline{c}) = \mu(\underline{c}) = \mu(\overline{c}) = 0. && \text{(A19)}
\end{aligned}$$

From (A18) and (A19), we have $\mu(\overline{c}) - \mu(\underline{c}) = \int_{\underline{c}}^{\overline{c}}[U_b'(W_\rho(c)) - \theta U_\rho'(\pi_\rho)]dF(c) = 0$. Therefore,

$$\theta = \frac{\int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(c))dF(c)}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(c))d(c)} > 0. \tag{A20}$$

Define $h(c) = \mu c + \lambda$. From condition (A16), on any interval where q is strictly decreasing, $h(c)$ and $\lambda(c)$ must be zero since both μc and λ are nonnegative. So $h'(c) = \mu + \mu' c + \lambda' = 0$, which leads to $\mu = -\mu' c - \lambda'$. Substituting (A17) and (A18) for μ' and λ', we have

$$\mu = \int_{\underline{c}}^{c} [U_b'(W_\rho(c)) - \theta U_\rho'(\pi_\rho)]dF(x) = U_b'(W_\rho(c))[V(q_\rho(c)) - c]f(c). \tag{A21}$$

Substituting (A20) into (A21) for θ, we have

$$U_b'(W_\rho(c))[V'(q)-c]f(c) = \int_{\underline{c}}^{c} \left[U_b'(W_\rho(x)) - \frac{\int_{\underline{c}}^{\overline{c}} U_b'(W_\rho(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)} U_\rho'(\pi_\rho(x)) \right] dF(x). \tag{A22}$$

Moreover, condition (A20) implies $\int_{\underline{c}}^{\overline{c}} U_\rho(\pi_\rho(c))dF(c) = \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))dF(c) = 0$.

7.3 Proof of Corollary 1

When the buyer is risk neutral, condition (A22) becomes

$$[V'(q_\rho) - c]f(c) = F(c) - \frac{\int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)}. \tag{A23}$$

Since $1 - U_\rho'(\pi(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ is strictly decreasing in c, its sign changes once and only once with $1 - U_\rho'(\pi_\rho(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x) > 0$ (< 0) at $c = \underline{c}$ ($c = \overline{c}$). Moreover, $F(c) - \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ equals zero at $\underline{c}$ and $\overline{c}$. Therefore, $F(c) - \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ must be positive on $(\underline{c}, \overline{c})$.

When c follows a uniform distribution, from (A23) we have

$$\frac{dV'(q_\rho)}{dc} = 2 - \frac{U_\rho'(\pi_\rho(c))}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)}. \tag{A24}$$

Since $U_\rho'(\pi_\rho(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x) < 1$ in the neighborhood of $\underline{c}$ and $U_\rho'(\pi_\rho(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ is strictly increasing in c, (A24) suggests $dV'(q_\rho)/dc > 0$ and the monotonicity condition is not constraining in the neighborhood of $\underline{c}$.

When the supplier converges to risk neutral, $U_\rho'(\pi_\rho(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ converges to

1. In that case, (A24) suggests that $dV'(q_\rho)/dc > 0$ and q_ρ is strictly decreasing on $[\underline{c}, \overline{c}]$.

For any monotonically decreasing schedule of $q_\rho(c)$ satisfying (A23), a lower bound for $U_\rho'(\pi_\rho(c))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ can be obtained by setting $q_\rho(c) = q_\rho(\overline{c})$ for all $c \in [\underline{c}, \overline{c}]$. It can be readily shown that the lower bound for $U_\rho'(\pi_\rho(\overline{c}))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)$ is greater than 2 and therefore $U_\rho'(\pi_\rho(\overline{c}))/\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x) > 2$ when the supplier is sufficiently risk averse. In that case, there exists some c' such that $dV'(q_\rho)/dc < 0$ and bunching occurs for the entire interval of $[c', \overline{c}]$.

For any bunching range $[c', \overline{c}]$, $\lambda(c') = 0$ for continuity. Therefore, from (A17) we have

$$\int_{c'}^{\overline{c}} [V'(q_\rho) - \frac{U_\rho'(\pi_\rho(c))c}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)}]dc = 0.$$

Moreover, $q_\rho(c')$ is determined by (A23).

7.4 Proof of Corollary 2

When the buyer is risk averse, $U_b'(W_\rho(c))$ is non-decreasing in c as $W_\rho(c)$ is non-increasing in c and $\partial U_b'(W_\rho(c))/\partial c = U_\rho''(W_\rho(c))W_\rho'(c) \geqslant 0$. In addition, since $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho$ equals 0 at $\underline{c}$ and equals 1 at $\overline{c}$, there must exist some cutoff point $\widehat{c} \in (\underline{c}, \overline{c})$ such that $U_b'(W_\rho(c))/B_\rho \geqslant 1$ if $c > \widehat{c}$ and $U_b'(W_\rho(c))/B_\rho \leqslant 1$ when $c < \widehat{c}$. Moreover, $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho$, the buyer's CE of the increase in ex post information rent as a result of raising $q_\rho(c)$ by one unit, equals $F(c)$ at $\underline{c}$ and $\overline{c}$, but is less than or equal to $F(c)$ for $c \in (\underline{c}, \overline{c})$.

When the buyer is not very risk averse, based on the proof of Corollary 1 and by continuity, $W_\rho(c)$ must be strictly decreasing in c and $F(c) > \int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho > \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho$ for $c \in (\underline{c}, \overline{c})$. Consequently, the properties in Corollary 1 continue to hold. That is, the production schedule is at the first-best level at $\underline{c}$ and $\overline{c}$ but below the first-best level for $c \in (\underline{c}, \overline{c})$ if the supplier is not very risk averse; bunching occurs in the production schedule for high cost realizations if the supplier is sufficiently risk averse.

As the buyer becomes more risk averse, everything else equal, $U_b'(W_\rho(x))$ increases more rapidly in c. Then, the difference in CEs of the ex post information rent between the two parties further decreases. In the extreme case where the buyer is infinitely risk averse, the optimal contract is a fixed price contract – the buyer charges a payment constant in c and the supplier produces the efficient level of output.

To prove that $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho \geqslant \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho$ for $c \in (\underline{c}, \overline{c})$ regardless how risk averse the supplier is, suppose $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho < \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx/S_\rho$ for $c \in (c_a, c_b)$ in the optimal contract and $\underline{c} < c_a < c_b < \overline{c}$. Then $q_\rho(c)$ is above the efficient level of output for $c \in (c_a, c_b)$. Define the efficient level of output for the cost realization c as $q_\rho^*(c)$ and construct a new contract by replacing the original contract $\{T_\rho(c), q_\rho(c)\}$ with $\left\{\widehat{T}_\rho(c), q_\rho^*(c)\right\}$ for $c \in (c_a, c_b)$ where $\widehat{T}_\rho(c) = T_\rho(c) - (q_\rho(c) - q_\rho^*(c))c$. Notice that under the constructed contract the supplier receives the same profit as in the original contract and the incentive compatibility constraint is not binding as the supplier produces a smaller output compared with the original contract. However, the constructed contract will outperform the original contract as the increase in production efficiency increases the buyer's expected utility.

Next we prove that there is less production distortion compared with the case of a risk neutral buyer. First, recall that $\int_{\underline{c}}^{c} U_b'(W_\rho(x))dF(x)/B_\rho$ equals $F(c)$ at $\underline{c}$ and $\overline{c}$, but is less than or equal to $F(c)$ for $c \in (\underline{c}, \overline{c})$ with a risk averse buyer. Then, everything else equal, the right-hand side of equation (8) becomes smaller as the buyer becomes risk averse. Moreover, since $U_b'(W_\rho(c))/B_\rho \geqslant 1$ for $c > \widehat{c}$, equation (8) suggests $V'(q_\rho(c)) - c$ is smaller for $c > \widehat{c}$ when the buyer becomes risk-averse.

Second, from equation (8),

$$
\begin{aligned}
V'(q_\rho(c)) - c &= \frac{\int_{\underline{c}}^{c} U_b'(W_\rho(x))dx}{U_b'(W_\rho(c))} - \frac{B_\rho \int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx}{S_\rho U_b'(W_\rho(c))} \\
&\leqslant F(c) - \frac{\int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dx}{S_\rho} \text{ for } c < \widehat{c}.
\end{aligned}
$$

as

$$
\frac{\int_{\underline{c}}^{c} U_b'(W_\rho(x))dx}{U_b'(W_\rho(c))} \leqslant F(c) \text{ and } \frac{B_\rho}{U_b'(W_\rho(c))} \geqslant 1
$$

for $c < \widehat{c}$.

With less downward production distortion, the monotonicity condition is less constraining and bunching occurs less often when the buyer becomes risk averse.

7.5 Proof of Proposition 3

The Hamiltonian is

$$
\begin{aligned}
H &= \{\alpha_l U_l(W_l(c)) + \alpha_h U_h(W_h(c))\} f(c) + \mu_l c z_l + \mu_h c z_h \\
&\quad + \lambda_l z_l + \lambda_h z_h + \theta U_h(\pi_h) f(c) + \beta[U_l(\pi_l) - U_l(\pi_h)] f(c),
\end{aligned} \tag{A25}
$$

where μ_l, μ_h, λ_l, λ_h, θ, and β are the Lagrange multipliers.

The necessary conditions are given by

$$\begin{aligned}
\frac{\partial H}{\partial z_l} &= \mu_l c + \lambda_l \geqslant 0,\ \ z_l \leqslant 0,\ \text{ and } (\mu_l c + \lambda_l) z_l = 0; && \text{(A26)}\\
\frac{\partial H}{\partial z_h} &= \mu_h c + \lambda_h \geqslant 0,\ \ z_h \leqslant 0,\ \text{ and } (\mu_h c + \lambda_h) z_h = 0; && \text{(A27)}\\
\lambda_l' &= -\frac{\partial H}{\partial q_l} = -[\alpha_l U_b'(W_l) V'(q_l) - \beta U_l'(\pi_l) c] f(c); && \text{(A28)}\\
\lambda_h' &= -\frac{\partial H}{\partial q_h} = -[\alpha_h U_b'(W_h) V'(q_h) - \theta U_h'(\pi_h) c + \beta U_l'(\pi_h) c] f(c); && \text{(A29)}\\
\mu_l' &= -\frac{\partial H}{\partial T_l} = -[-\alpha_l U_b'(W_l) + \beta U_l'(\pi_l)] f(c); && \text{(A30)}\\
\mu_h' &= -\frac{\partial H}{\partial T_h} = -[-\alpha_h U_b'(W_h) + \theta U_h'(\pi_h) - \beta U_l'(\pi_h)] f(c);\ \text{and} && \text{(A31)}\\
\lambda_\rho(\underline{c}) &= \lambda_\rho(\overline{c}) = \mu_\rho(\underline{c}) = \mu_\rho(\overline{c}) = 0, \text{ where } \rho = l, h. && \text{(A32)}
\end{aligned}$$

From the transversality condition (A32) and equation (A30),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha_l U_b'(W_l) - \beta U_l'(\pi_l)] dF(c) = 0, \tag{A33}$$

which provides

$$\beta = \frac{\alpha_l \int_{\underline{c}}^{\overline{c}} U_b'(W_l) dF(c)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(c)}. \tag{A34}$$

From the transversality condition (A32) and equation (A31),

$$\mu_h(\overline{c}) - \mu_h(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha_h U_b'(W_h) - \theta U_h'(\pi_h) + \beta U_l'(\pi_h)] dF(c) = 0, \tag{A35}$$

which provides

$$\theta = \frac{\alpha_h \int_{\underline{c}}^{\overline{c}} U_b'(W_h) dF(c)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(c)) dF(c)} - \frac{\alpha_l \int_{\underline{c}}^{\overline{c}} U_b'(W_l) dF(c) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(c)) dF(c)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(c)) dF(c) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(c)) dF(c)}. \tag{A36}$$

When q_ρ is strictly decreasing in c, we have

$$h_\rho'(c) = \mu_\rho + \mu_\rho' c + \lambda_\rho' = 0 \text{ or } \mu_\rho = -\mu_\rho' c - \lambda_\rho'. \tag{A37}$$

Then substituting μ_ρ' and λ_ρ' into (A37), we have

$$\begin{aligned} \mu_l &= \int_{\underline{c}}^{c} [\alpha_l U_b'(W_l) - \beta U_l'(\pi_l)] dF(x) \qquad \text{(A38)} \\ &= \int_{\underline{c}}^{c} [\alpha_l U_b'(W_l) - \frac{\alpha_l U_l'(\pi_l) \int_{\underline{c}}^{\overline{c}} U_b'(W_l) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(x)}] dF(x) \\ &= \alpha_l U_b'(W_l)[V'(q_l(c)) - c] f(c), \text{ and} \end{aligned}$$

$$\begin{aligned} \mu_h &= \int_{\underline{c}}^{c} [\alpha_h U_b'(W_h) - \theta U_h'(\pi_h) + \beta U_l'(\pi_h)] dF(x) \qquad \text{(A39)} \\ &= \alpha_h \int_{\underline{c}}^{\overline{c}} U_b'(W_h) dF(x) \left[\frac{\int_{\underline{c}}^{c} U_b'(W_h(x)) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_b'(W_h(x)) dF(x)} - D_h(c) \right] + \alpha_l G(c) \int_{\underline{c}}^{\overline{c}} U_b'(W_l) dF(x) \\ &= \alpha_h U_b'(W_h)[V'(q_h(c)) - c]. \end{aligned}$$

7.6 Proof of Corollary 3

Since

$$G(c) \equiv \frac{\int_{\underline{c}}^{c} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)}, \tag{A40}$$

we have

$$G'(c) = \frac{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} R(c) f(c), \text{ where} \tag{A41}$$

$$R(c) = \left[\frac{U_l'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)} - \frac{U_h'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)} \right]. \tag{A42}$$

Since $U_h(\cdot)$ must be a strictly concave transformation of $U_l(\cdot)$, there exists a strictly concave function $Y(\cdot)$ such that $U_h(\cdot) \equiv Y(U_l(\cdot))$. Therefore,

$$R(c) = \left[\frac{1}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)} - \frac{Y'(U_l(\pi_h(c)))}{\int_{\underline{c}}^{\overline{c}} Y'(U_l(\pi_h(x)))U_l'(\pi_h(x))dF(x)} \right] U_l'(\pi_h(c)). \tag{A43}$$

Notice that $\pi_h(c)$ is strictly decreasing in c, consequently $Y'(U_l(\pi_h(c)))$ is strictly increasing in c. Moreover, $G(\underline{c}) = G(\overline{c}) = 0$. Therefore, there exists some c_0 such that $R(c) > 0$ in $[\underline{c}, c_0)$ and $R(c) < 0$ in $(c_0, \overline{c}]$; and $G(c) > 0$ on $(\underline{c}, \overline{c})$.

7.7 Proof of Corollary 4

With a uniform distribution, we have $f'(c) = 0$. In the case of common information on risk preference, (A9) provides

$$\frac{dV'(q_h(c))}{dc} = 2 - \frac{U_h'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)}. \tag{A44}$$

As shown in the proof of Proposition 1, the right-hand side of (A44) is positive at $c = \underline{c}$ and is strictly decreasing in c. Therefore, in the case of common information on risk preference, bunching $(dV'(q_h(c))/dc < 0)$ can occur only in some interval $[c', \overline{c}]$.

In the case of asymmetric information on risk preference, from (A39) we have

$$\frac{dV'(q_h(c))}{dc} = \left[2 - \frac{U_h'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)}\right] + \frac{1-\alpha}{\alpha}G'(c). \tag{A45}$$

Notice that the second therm on the right-hand side of (A45) shows the effect of asymmetric information on risk preference. As shown in the proof of Corollary 3, there exists some c_0 such that $G'(c) > 0$ in $[\underline{c}, c_0)$ and $G'(c) < 0$ in $(c_0, \overline{c}]$.

Therefore, if bunching $(dV'(q_h(c))/dc < 0)$ occurs in the neighborhood of $\overline{c}$ in the case of common information, it must also occurs in the case of asymmetric information on risk preference. However, if bunching $(dV'(q_h(c))/dc < 0)$ occurs in the neighborhood of $\overline{c}$ in the latter case, it may not occur in the former case.

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